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THE INVESTIGATION OF CONDITIONS OF THE EXTREME ARRANGEMENT OF SEVERAL CLASSICAL GEOMETRIC FIGURES WITH A COMMON CENTER BY ESTIMATING THE LENGTH OF THE LINE AND THE AREA OF THEIR DIVERGENCE

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Abstract

The article deals with the question of the optimal, extreme (minimum) location of one flat figure, namely, a square in the first case, as well as an equilateral triangle in the second case relative to a circle with a common center of these figures. The main criterion for the optimality of such mutual placement of one figure relative to another are such criteria as the effective location of the total length of the set of lines, according to which the discrepancy of these figures and the estimate of the area of their discrepancy occur. The value of a certain function that determines the length of the sum of the lines of discrepancy of the figures in both cases and the function that determines the area of discrepancy is obtained. A study of the extremality of such functions has been performed and it has been shown that at the found point of the extremum, the function determines the total length of the line of divergence of the figures and the corresponding area of divergence acquires extreme values. Figures are given for a better understanding of the formulation and solution of the problem. Conclusions are made in which the values of the required arguments are given and the corresponding function will acquire extreme values.

Keywords: circle, square, equilateral triangle, length of the arc of a circle and its part, length of the side of the square, area, extremality of functions of one variable.

Introduction. Relevant both from the point of view of theory and from a number of specific practical applications is the problem of effective extreme (minimum) location of one flat geometric figure relative to another [1-8], in particular, the location of a circle relative to a square (Fig. 1) and an equilateral triangle (Fig. 2). It is established that only at certain ratios of the radius of the circle to the length of the side of the square or triangle, the length of the arc of the total line, which is the discrepancy of these geometric objects, will acquire extreme (minimum) value. The results of the study are devoted to the search for such mutual relations of the mentioned geometrical characteristics of these figures.

Materials and methods When solving a number of applied technical problems to study the optimal extreme values, the standard mathematical approach [1-8] is most often used, according to this approch the value of some function depending on one or more variables is established, and the corresponding conditions (equations or system of equations), from which it is possible to establish the extreme values of such arguments and, finally, the value of the function introduced into consideration when the values of the extreme arguments are found. The article continues the analysis of studies of the mutual effective arrangement of standard geometric figures, namely, the effective location of the arc of a circle relative to first a square, then an equilateral triangle with a common center of these figures, as it has been studied in the scientific work [1]. But if in the scientific work [1] the function, which is based on the establishment of a certain area, for which the geometric figures do not coincide, has been calculated and studied, then in these study as the basic working function offers another numerical characteristic, such as some length of the total polyline or arcs, etc..

As the main criterion for such an effective coverage of one figure with another with a common center, it is proposed to choose such an indicator as the minimum length of the line [1,2,6], on which these geometric figures do not match. On the basis of this geometric indicator the corresponding function depending on a certain argument is constructed and the research of this function by methods of differential calculus is carried out. It is shown that this function acquires the minimum values which are established at extreme values of this argument.

Experimental 1. Supposing that we have a square with the length of its side equal to $2a$ (Fig. 1).

That is, $OA = a, OM = ON = r$ is the radius of a circle centered at point O. This point is also the center of the entered square. φ is the value of the desired optimal angle between the leg OA and the hypotenuse ON, which depends on the ratio between the length of the side of the square and the radius of the circle to obtain the optimal (minimum) value of the function that characterizes the length of the total line. Let the length L be the total value of the lengths L_1 and L_2within the first 45^0 of the first quarter, where the figures indicated at the beginning of the work do not coincide (have a discrepancy). Obviously, due to the symmetry of the figure, the total length of the line of divergence L_{full} will be equal to $8L$.

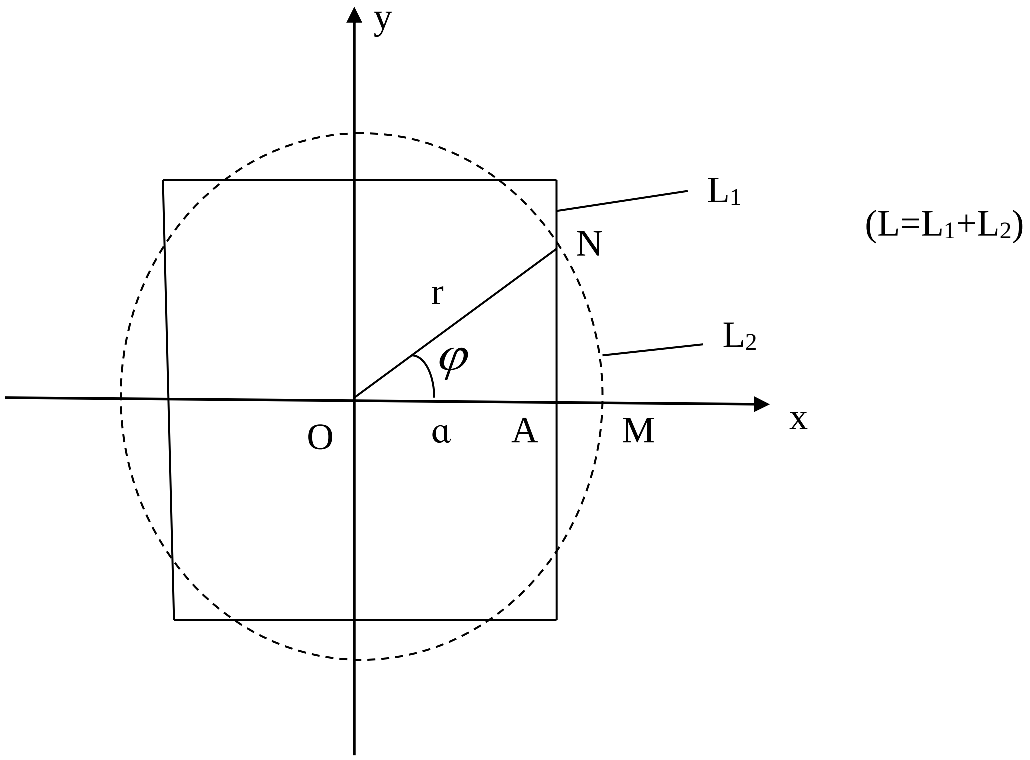

Fig.1. Covering a circle with a square with a common center, determining the unknown argument φ , which depends on the length of the line L , on which these figures do not match.

From the right triangle OAN we have obvious trigonometric relations:

$$\cos\varphi = \frac{a}{r} \Rightarrow a = r\cos\varphi. \quad (1)$$

For the first half of the first quarter we have a limit on the angle φ:

$$\varphi \in [0; 45^0]. \quad (2)$$

We also have: $L_1 = a - r sin\varphi = r(cos\varphi - \sin\varphi)$, (3)

$$L_2 = r\varphi, \quad (4)$$

$$L = L_1 + L_2 = r\varphi + r(cos\varphi - \sin\varphi) = \\ = r(\varphi + cos\varphi - \sin\varphi). \quad (5)$$

Here L is a function that establishes the total length of the arc of divergence of geometric figures. Finding the first derivative of this function by its argument φ and equating this value to 0, we obtain the equation

$$L'_\varphi = r(1 - \sin\varphi - \cos\varphi) = 0, \quad (6.1)$$

or: $\sqrt{2}\cos(\varphi - 45^0) = 1.$ (6.2)

Taking into account condition (2), we have a critical value of the angle $\varphi = 0$.

Find the second derivative of the function L and set its sign at $\varphi = 0$:

$$L''_{\varphi\varphi} = r(-\cos\varphi + \sin\varphi),$$

$$L''(\varphi = 0) = r(-\cos 0 + \sin 0) = -r < 0 \Rightarrow L \to max \quad (7)$$

For $\varphi = 0$ we obtain the ratio between the geometric characteristics of the radius of the circle and half the length of the side of the square:

$$a = r\cos 0 \Rightarrow a = r \quad (8)$$

This means that in the case of a circle inscribed in a square (Fig. 2), the value of L, and hence the total length of the outer line along which the geometric figures diverge, acquires the maximum value:

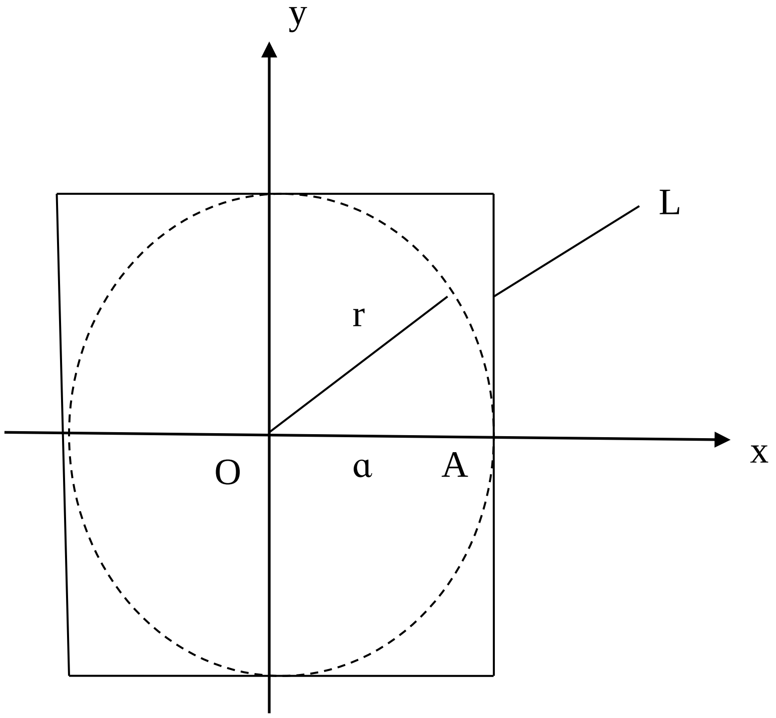

Fig.2. Inscribed in a square circle as the maximum value of the outer line of divergence of the geometric figures when $\varphi = 0$.

$$L_{full\,max}(\varphi = 0) = 8L = 8a = 8r. \tag{9}$$

Another extreme situation - if the square is inscribed in a circle (Fig. 3). In this case

$$L_{full\,min}(\varphi = 45^0) = 2\sqrt{2}a = 2\pi r. \tag{10}$$

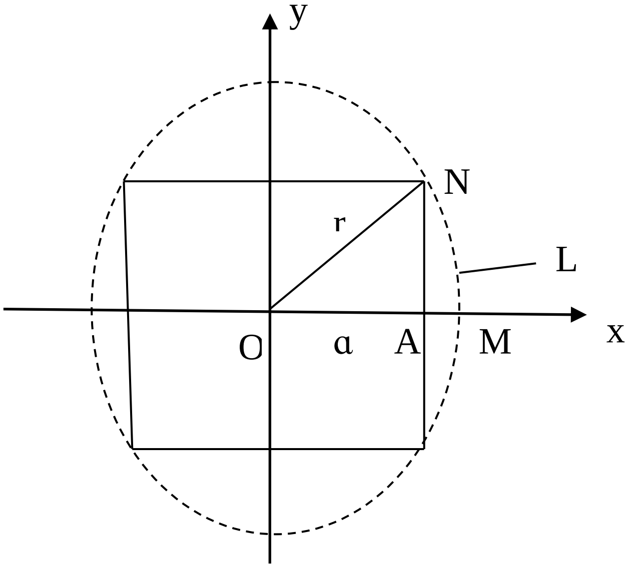

Fig.3. The square inscribed in a circle as the minimum minimum value of the outer line of divergence of geometric figures at $\varphi = 45^0$.

2. In this paragraph the problem similar to the one considered in the previous point 1 is solved and investigated, with the difference that the condition of an effective arrangement of an arc of a circle concerning an equilateral triangle with the common center of these figures is established (fig. 4). Let ABC be a triangle with side a whose center coincides with the plane of the circle of radius r. We have the following geometrical characteristics of the lengths of the corresponding segments:

$$\mathrm{OH} = r,\ \mathrm{O}R = \frac{a\sqrt{3}}{6},\ RH = \sqrt{r^2 - \frac{a^2}{12}},$$

$$HC = \frac{a}{2} - \sqrt{r^2 - \frac{a^2}{12}}, \tag{11}$$

$$\cos\beta = \frac{a\sqrt{3}}{6r} \Rightarrow r = \frac{a\sqrt{3}}{6\cos\beta}, \beta \in [0; 60^0]. \tag{12}$$

We can introduce the function L, which sets the total length of the line of divergence of the geometric figures relative to one of the vertices of the triangle, namely the vertex C, and this function is defined as follows:

$$L = 2TC + \cup\, TP = 2\left(\frac{a}{2} - \sqrt{r^2 - \frac{a^2}{12}}\right) + 2r\beta, \text{або} \quad (13.1)$$

$$L = \left(a - 2\sqrt{r^2 - \frac{a^2}{12}}\right) + 2r arccos\frac{a\sqrt{3}}{6r}. \quad (13.2)$$

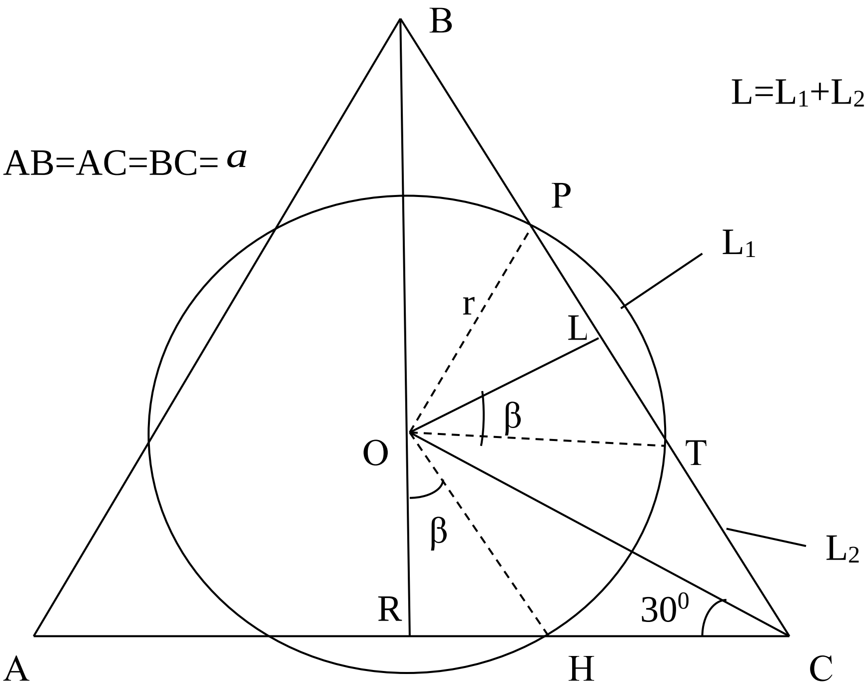

Fig. 4. Covering the circle with an equilateral triangle ABC with a common center O, determining the unknown argument β, which depends on the length of the line L, on which these figures do not match.

In this problem it is useful to introduce the function l reduced to L, namely:

$$l = \frac{L}{r} = \frac{a}{r} - 2\sqrt{1 - \frac{a^2}{12r^2}} + 2arccos\frac{a\sqrt{3}}{6r}. \quad (14.1)$$

Let $t = \frac{a}{r}$ be the only argument of the function l that depends on the fraction of the relation a to r, then with respect to l we have:

$$l = t - 2\sqrt{1 - \frac{t^2}{12}} + 2arccos\frac{t\sqrt{3}}{6}. \quad (14.2)$$

Find and equate to 0 the first derivative l:

$$l'_t = 1 + \frac{t - 2\sqrt{3}}{6\sqrt{1 - \frac{t^2}{12}}} = 0. \quad (15.1)$$

The obtained equation (14) with respect to the unknown argument t is written in the form:

$$6^2\left(1 - \frac{t^2}{12}\right) = (2\sqrt{3 - t})^2. \quad (15.2)$$

Where $t^2 - \sqrt{3}t - 6 = 0 \Rightarrow D = 27, \sqrt{D} = 3\sqrt{3} \Rightarrow t = 2\sqrt{3}$, (here t is the only logical solution of equation (15.2), since $t > 0$). At this value of t according to relation (12) we have the equation $\cos\beta = 1$cosβ or $\beta = 0$. The answer of the required angle β means due to the necessary condition of existence of extreme values that on the interval $\beta \in [0; 60^0]$ the function l (and hence L) is monotonic depending on its argument, and therefore the smallest and largest of its possible values are reached at the ends of the corresponding segment from the specified area of definition of these functions. At $\beta = 0$ we have a situation of reaching the minimum of the function l, which is geometrically interpreted by the arrangement of the initial figures as a circle inscribed in an equilateral triangle (Fig. 5).

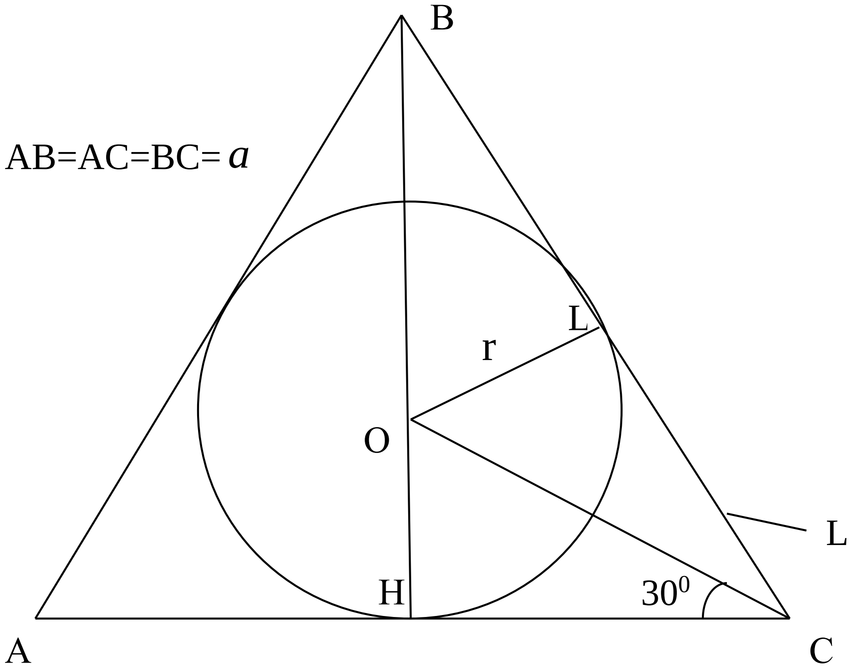

Fig.5. The limit position of geometric figures as inscribed in an equilateral triangle circle at the value of the parameter $\beta = 0$, for which the function L reaches its minimum value.

In this case ($\beta = 0$) the total length of the line that determines the discrepancy of the given geometric shapes is equal to:

$$L_{full\ min} = L_{\Delta ABC} = 3a = \frac{18}{\sqrt{3}}r. \qquad (16)$$

Another limiting, but already the maximum value of the total length of the line of divergence is achieved when the value of the parameter β equal to 60^0. In this case, geometrically we have the situation when the arc of a circle describes a given equilateral triangle (Fig. 6). Thus the full length of an external line on which the specified geometrical curves run, obviously, will coincide with the length of an arc of all circle:

$$L_{full\ max} = 2\pi r = \frac{2\pi}{\sqrt{3}}a. \qquad (17)$$

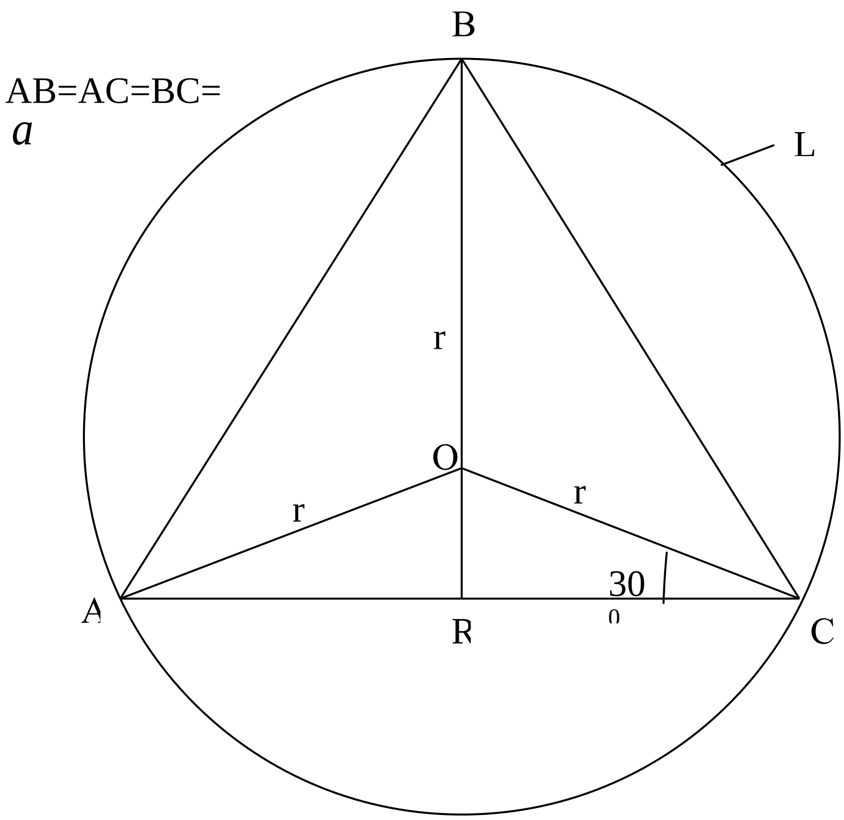

Fig. 6. The limiting arrangement of geometric figures as described around an equilateral triangle is a circle about the center at the point O at the value of the parameter $\beta = 60^0$, for which the function L reaches the maximum value.

The function can be proposed as another alternative criterion for the extreme relative position of several geometric figures with a common center. It sets the value of the total area by which these figures do not match. As an example of a specific implementation of this approach, consider the question of the effective placement of the area of a circle and the area of an equilateral triangle with a common center of these geometric shapes.

Supposing that we have an equilateral triangle ABC with side α, whose area is covered by another area, namely, the area of the circle of radius r with the common center of these two figures (Fig. 7).

We have OH = OT = OP = r, AB = AC = BC = α. The points H, T, P can vary depending on the change in the radius of the circle itself or on the change of the length of the side of an equilateral triangle. Therefore, the angle β, which is the angle between the OR catheter and the hypotenuse OH of the right-angled triangle ORN, will also be a variable. The limits from which the specified angle β can be changed, namely: $0 < \beta < 60^0$, are obvious from (Fig.7), since the angle ^ ROH = 60^0.

In the entered notation there are obvious trigonometric ratios:

$$\cos\beta = \frac{a\sqrt{3}}{6r} \Rightarrow \frac{a}{r} = \frac{6}{\sqrt{3}}\cos\beta = 2\sqrt{3}\cos\beta. \tag{18}$$

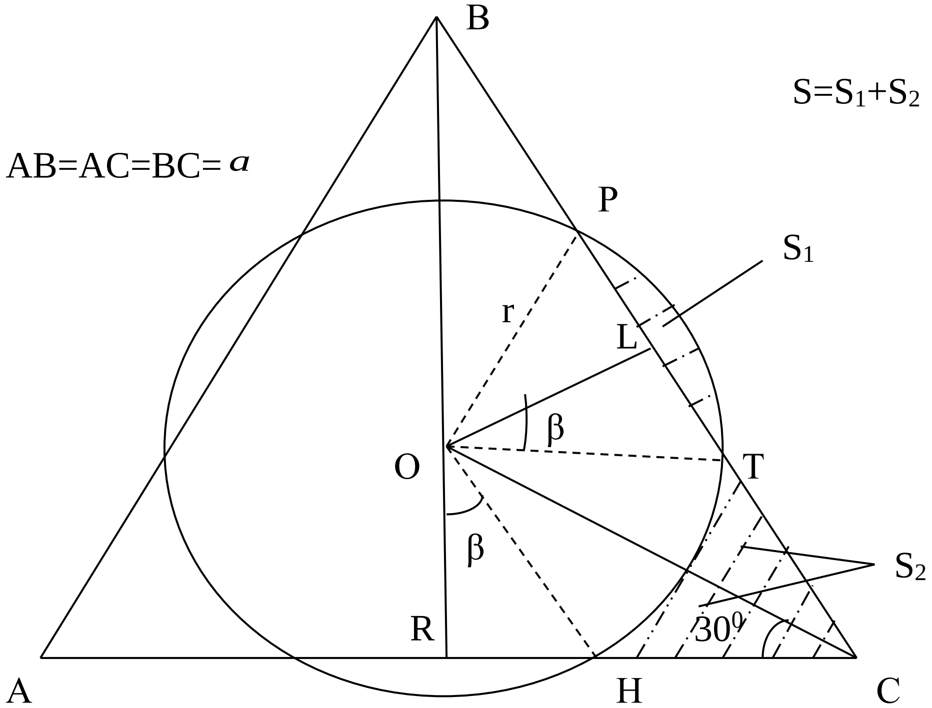

Fig. 7. Equilateral triangle ABC with side α, whose area is covered by the area of the circle of radius r with common center.

As part of the area on which the geometric figures do not coincide, as shown in (Fig.7), we take the area $S = S_1 + S_2$. Then, obviously, the total area over which the figures do not coincide will be 3S. For the components of the additions of the required area using the appropriate geometric formulas we have the following equations:

$$S_1 = \frac{1}{2}r^2(2\beta) - \frac{1}{2}OL \cdot TP = r^2\beta - \frac{a\sqrt{3}}{12}(a - 4r\sin(60^0 - \beta)). \tag{19}$$

$$S_2 = 2(\frac{1}{2}OC \cdot HC\sin 30^0 - \frac{1}{2}r^2(\frac{\pi}{3} - \beta)) = \frac{2\sqrt{3}}{3}ar\sin(60^0 - \beta) - r^2\frac{\pi}{3} + r^2\beta. \tag{20}$$

Thus, the total area becomes significant

$$\begin{aligned} S = S_1 + S_2 &= r^2\beta - \frac{a\sqrt{3}}{12}(a - 4r\sin(60^0 - \beta)) + \frac{2\sqrt{3}}{3}ar\sin(60^0 - \beta) - r^2\frac{\pi}{3} + r^2\beta = \\ &2r^2\beta - \frac{a^2\sqrt{3}}{12} + \sqrt{3}ar\sin(60^0 - \beta) - r^2\frac{\pi}{3}. \end{aligned} \tag{21}$$

Conducting in the last equality identical algebraic transformations using relation (18), we obtain the final form of the function of the variable area S using the formula (22), which is definitive for further study:

$$S=r^2(2\beta-\frac{\sqrt{3}}{12}\left(\frac{a}{r}\right)^2+\sqrt{3}\frac{a}{r}\sin(60^0-\beta)-\frac{\pi}{3})=r^2(2\beta-\frac{\sqrt{3}}{12}\left(2\sqrt{3}cos\beta\right)^2+$$
$$\sqrt{3}\cdot(2\sqrt{3}\cos\beta)\sin(60^0-\beta)-\frac{\pi}{3})=r^2(2\beta+\sqrt{3}-\frac{\pi}{3}+\sqrt{3}cos2\beta-\frac{3}{2}\sin 2\beta). \quad (22)$$

In this last relation (22) we have a record of a function S dependent on the variable argument β. Thus, it remains to investigate for the existence of extreme values the obtained function.

Therefore, we first find the value of the first derivative of the function obtained by its argument β and equate the value of this derivative to zero:

$$S'_\beta=r^2(2-2\sqrt{3}\sin 2\beta-3\cos 2\beta)=0. \quad (23)$$

From the obtained equation (6) we have the following equality with respect to the unknown argument β:

$$4\sqrt{3}\sin\beta\cos\beta+3\cos^2\beta-3\sin^2\beta-2\sin^2\beta-2\cos^2\beta=0. \quad (24)$$

Using known trigonometric transformations, equality (7) is transformed into the following:

$$5tg^2\beta-4\sqrt{3}tg\beta-1=0 \quad (25)$$

Solve the obtained quadratic equation with respect to:

Then we have unknown solutions of equation (25) are as follows:

$$(tg\beta)_1=\frac{\sqrt{17}-2\sqrt{3}}{5},$$
$$(tg\beta)_2=\frac{\sqrt{17}+2\sqrt{3}}{5}.$$

Since $tg60^0=\sqrt{3}\approx 1{,}73$, therefore, both values will be within the range of possible values of β.

The nature of the possible extreme values of the angle β is found by finding and investigating the second derivative of S. We have:

$$S''_{\beta\beta}=r^2(-4\sqrt{3}cos2\beta+6\sin 2\beta)=r^2(12\sin\beta\cos\beta-4\sqrt{3}cos^2\beta+4\sqrt{3}\sin^2\beta)=$$
$$=\frac{r^2}{cos^2\beta}(\sqrt{3}tg^2\beta+3tg\beta-\sqrt{3}). \quad (26)$$

Find the signs of the second-order derivative at both points $\beta_{1,2}$ of the possible extremum of the function under study:

$$S''_{\beta\beta}((tg\beta)_1)=\frac{4r^2}{cos^2\beta}(\sqrt{3}\left(\frac{\sqrt{17}-2\sqrt{3}}{5}\right)^2+3\left(\frac{\sqrt{17}-2\sqrt{3}}{5}\right)-\sqrt{3})<0, \quad (27.1)$$

$$S''_{\beta\beta}((tg\beta)_2)=\frac{4r^2}{cos^2\beta}(\sqrt{3}\left(\frac{\sqrt{17}+2\sqrt{3}}{5}\right)^2+3\left(\frac{\sqrt{17}+2\sqrt{3}}{5}\right)-\sqrt{3})>0. \quad (27.2)$$

Since the value of the second derivative of function S at point β_1 becomes negative, therefore, at this point, the function S takes the largest of its possible values, at the point β_2, the second derivative is positive, so at this point the function S takes the smallest of its possible values (Fig. 8). Note that $0<\beta_{1,2}<60^0$.

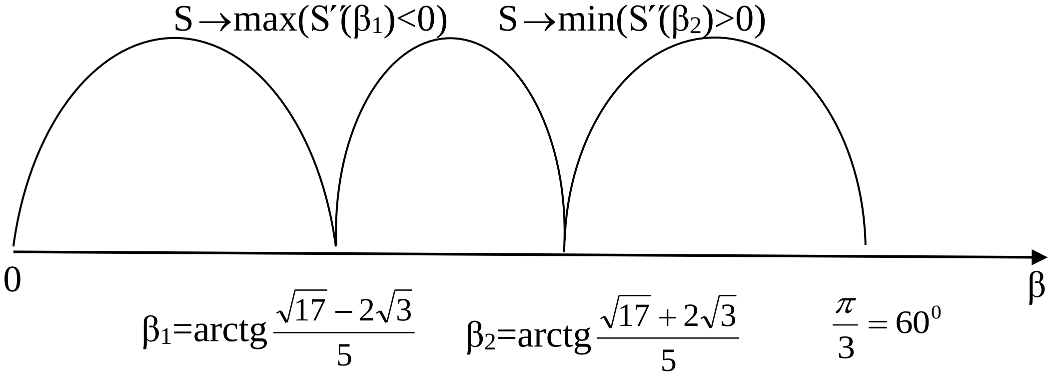

Fig.8. Investigation of the extreme values character of the angle β by the results of the second derivative of S.

Setting the ratio of the parameters of the geometric characteristics of the side of an equilateral triangle α and the radius of the circle r, this ratio at each of the extremal points will be as follows:

$$\frac{a}{r}=2\sqrt{3}\cos\beta\Big|_{\beta=\beta_1}=\frac{2\sqrt{3}}{\frac{1}{\cos\beta_1}}=\frac{2\sqrt{3}}{\sqrt{1+\left(\frac{\sqrt{17}-2\sqrt{3}}{5}\right)^2}} \tag{28.1}$$

with $S\to\max$, and the ratio

$$\frac{a}{r}=2\sqrt{3}\cos\beta\Big|_{\beta=\beta_{21}}=\frac{2\sqrt{3}}{\frac{1}{\cos\beta_2}}=\frac{2\sqrt{3}}{\sqrt{1+\left(\frac{\sqrt{17}+2\sqrt{3}}{5}\right)^2}} \tag{28.2}$$

with $S\to\min$.

The total cumulative area over which the figures discussed in this paper will not be three times larger. To set the numerical values of the smallest and largest such areas, respectively, in each of the extreme values of the angle β for simplifications, we introduce the following notation:

$$N_1=\frac{\sqrt{17}-2\sqrt{3}}{5},\ N_2=\frac{\sqrt{17}+2\sqrt{3}}{5}.$$

Then according to the angle $\beta=\beta_1$ using equality (28.1) we have:

$$\cos\beta_1=\frac{1}{\sqrt{1+N_1^2}},\ \sin\beta_1=\frac{N_1}{\sqrt{1+N_1^2}},$$

and then the total largest area with the result (22) takes the form:

$$S_{\max}=3S(\beta_1)=3r^2(2arctgN_1+\sqrt{3}-\frac{\pi}{3}+\sqrt{3}(\frac{1}{1+N_1^2}-\frac{2N_1}{1+N_1^2})-\frac{3}{2}\cdot 2\frac{N_1}{\sqrt{1+N_1^2}}\cdot$$
$$\cdot\frac{1}{\sqrt{1+N_1^2}})=3r^2(2arctgN_1+\sqrt{3}-\frac{\pi}{3}+\frac{\sqrt{3}}{1+N_1^2}-(3+2\sqrt{3})\frac{N_1}{1+N_1^2}). \tag{29}$$

Similarly, when $\beta=\beta_2$ and

$$\cos\beta_2=\frac{1}{\sqrt{1+N_2^2}},\ \sin\beta_2=\frac{N_2}{\sqrt{1+N_2^2}},$$

then we have the total, taking into account the result (22), the smallest area:

$$S_{\min}=3S(\beta_2)=3r^2(2arctgN_2+\sqrt{3}-\frac{\pi}{3}+\sqrt{3}(\frac{1}{1+N_2^2}-\frac{2N_2}{1+N_2^2})-\frac{3}{2}\cdot 2\frac{N_2}{\sqrt{1+N_2^2}}\cdot$$
$$\cdot\frac{1}{\sqrt{1+N_2^2}})=3r^2(2arctgN_2+\sqrt{3}-\frac{\pi}{3}+\frac{\sqrt{3}}{1+N_2^2}-(3+2\sqrt{3})\frac{N_2}{1+N_2^2}). \tag{30}$$

Using equality (18), we can write equality (29) and (30) in equivalent form by applying the length of side α of an equilateral triangle (these relations are not given).

Conclusions. The article investigates the question of the optimal effective mutual arrangement of two geometric figures with a common center, namely the arc of a circle, first relative to a square and then relative to an equilateral triangle. As the main criterion for such mutual arrangements, it is proposed to take the length of the total line along which such figures diverge (outer line of divergence). The study of the function, which establishes the entire length of the line along which the discrepancy of the figures is observed, the extreme indicates that this function acquires minimal values in the case of such an arrangement of figures when the circle is inscribed in a square or equilateral triangle (Fig. 2, Fig. 5).), and when the circle described around these figures (Fig. 3, Fig. 6) such a function is maximal. The results of the study on the one hand are purely theoretically classical, on the other hand, these results can be used in some transport problems for efficiency, in solving agronomic issues of optimal land management, and so on.

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