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MATHEMATICAL MODEL OF THE EPIDEMIC DEVELOPMENT

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АНОТАЦІЯ

В статті розглянуто питання побудови математичної моделі розвитку абстрактної епідемії за визначених початкових умов. Відзначено що епідемії супроводжують людство на протязі всього його існування. Епідемії і пандемії виступають як один з дієвих факторів регулювання кількості населення на Землі. На-жаль всі досягнення сучасної медицини не можуть перешкодити виникненню і розповсюдженню старих і нових хвороб. Для боротьби з ними потрібен системний підхід , який передбачає, в тому числі, і створення математичних моделей які описують динаміку розповсюдження епідемій. Звичайно, складність проблеми (неоднозначно виражені початкові умови, стохастичний характер перебігу подій тощо) не дозволяють створити універсальну математичну модель яка б адекватно описувала динаміку перебігу епідемії. Перші спроби створення математичних моделей для опису перебігу епідемій були зроблені ще в XIX сторіччі в Англії. Звичайно, з сучасної точки зору вони були недосконалими, не враховували багато життєвих факторів. В подальшому, з розвитком комп'ютерних технологій, моделі ставали все більш досконалими. Але, як відзначають сучасні дослідники, наразі ще не створена універсальна математична модель розвитку епідемій.

В даній статті розглядається окремий випадок зі своїми допущеннями і початковими умовами. Вся популяція людей умовно поділена на три групи: перша – особи сприйнятливі до хвороби, але поки що здорові; друга – інфіковані особи які самі хворі і є джерелом розповсюдження епідемій; третя – здорові люди які мають імунітет до даної хвороби. Зроблені припущення що швидкість зміни сприйнятливих до хвороби людей пропорційна початковому числу сприйнятливих осіб, а також що швидкість зміни інфікованих але видужуючих осіб буде пропорційна числу інфікованих людей. Також зроблено припущення що коефіцієнти захворюваності і видужування однакові. Для даних умов складені та вирішені диференційні рівняння і на їх основі побудовані графічні залежності які показують динаміку зміни числа осіб в кожній із трьох груп людей. Отриманий вираз для визначення часу при якому кількість інфікованих осіб буде максимальна. В цей момент часу кількість сприятливих до хвороби осіб співпадає з числом інфекційних людей. Отримано рівняння для прогнозу часу завершення епідемії. Він залежить від співвідношення осіб в першій та другій групах. Побудовані графіки зміни в часі кількості осіб в усіх трьох групах.

ABSTRACT

The article deals with the problem of constructing a mathematical model for the development of an abstract epidemic under certain initial conditions. It has been noted that epidemics accompany humanity throughout its existence. Epidemics and pandemics act as one of the effective factors in regulating population size on Earth. Unfortunately, all the achievements of modern medicine cannot prevent the emergence and spread of old and new diseases. To combat them requires a systematic approach that involves, inter alia, the creation of mathematical models that describe the dynamics of epidemics. Of course, the complexity of the problem (ambiguous initial conditions, stochastic nature of the events, etc.) do not allow to create a universal mathematical model that would adequately describe the dynamics of the epidemic. The first attempts to create mathematical models to describe the course of epidemics were made in the nineteenth century in England. Of course, from the modern point of view they were imperfect, they did not take into account many factors of life. Subsequently, with the development of computer technology, models have become more sophisticated. But, as today's researchers point out, a universal mathematical model for epidemic development has not yet been created.

This article deals with a specific case with its assumptions and initial conditions. The whole population of people is conditionally divided into three groups: the first is persons susceptible to the disease, but so far healthy; the second is infected persons who are ill and are the source of epidemics; third - healthy people who are immune to this disease. The assumption is made that the rate of change of susceptible people is proportional to the initial number of susceptible persons, and that the rate of change of infected but recovering persons will be proportional to the number of infected people. It has also been suggested that the morbidity and recovery rates are the same. For these conditions, differential equations are drawn up and solved and graphical dependencies are constructed on their basis, which show the dynamics of change in the number of persons in each of the three groups of people. An expression was obtained to determine the time at which the number of infected persons would be maximal. At this point in time, the number of people favorable for the disease coincides with the number of infectious people. An equation was obtained to predict the timing of the epidemic. It depends on the ratio of people in the first and second groups. Graphs of time changes in the number of persons in all three groups were constructed.

Ключові слова: математична модель, епідемія, інфекція, диференційне рівняння, імунітет, початкові умови.

Keywords: mathematical model, epidemic, infection, differential equation, immunity, initial conditions.

Formulation of the problem. Throughout life together with the person there are various diseases. The actual and average life expectancy of a person is much less than the potential laid down in it, largely due to illness. Out of all known diseases, epidemics are of particular importance to humans, that is, diseases that are new to the population and spread at a rate far exceeding that expected. Until recently, epidemics and pandemics have been the most effective factors in regulating population size (since they are derived from the Greek "demos" – the people). Despite all the advances in medicine, epidemics and pandemics continue to threaten humanity. There are no longer enough medical measures and remedies to combat these phenomena. At present, the magnitude and diversity of new and "old" epidemics requires a systematic approach to studying the causes of their occurrence and spread. And here is the basic science of humanity – mathematics, which helps to describe the course of the epidemic, the dynamics of its growth and attenuation, and so on.

Of course, the complexity and versatility of the problem does not allow to make a full forecast of the situation. That is, at the moment there is no universal model that adequately describes all aspects of the epidemic. It should also be borne in mind that the epidemic's development processes are highly dynamic. Therefore, the issues of modeling the dynamics of the epidemic are currently very relevant.

In this paper an attempt is made to build a mathematical model of the dynamics of epidemic development with a number of limitations and assumptions that reproduce the real life situation.

Analysis of recent research and publications. Attempts to simulate the process of the epidemic were first made in the 19-th century in England [1]. However, these models were relatively simple and did not take into account the stochastic nature of the processes they described. Further development of epidemic modeling was obtained in the second half of the 20-th century in the works of scientists of the USSR [2; 3]. Random factors of epidemic processes have already been taken into account in these works, but they were rather abstract in nature and little account was taken of the realities of epidemics.

A detailed analysis of the experience of creating mathematical models for forecasting the epidemic on the territory of the USSR and the Russian Federation is made in [4]. The paper notes that mathematical computer-aided prediction of influenza outbreaks for large territories was realized due to the following three conditions:

– was developed a mathematical model of the influenza epidemic;

– was constructed a matrix of passenger flows for the 100 largest cities of the USSR;

– was established an epidemic surveillance system throughout the country.

In [5] it was noted that the most common approximate (rough) models for describing the epidemic are linear, exponential and logistic. The first two models adequately describe well-studied processes in a limited space of input quantities. When modeling the situation with several stages of the life cycle, the S - like logistic model is more adequate [6]. Almost all authors note the presence of various difficulties in the construction of models due to the inability to take into account all the factors that affect the process and their great stochasticity.

For example, in [7] the statistical characteristics of epidemic processes were evaluated, and it was noted that the variations in the numbers characterizing the process can reach significant values, and this should be taken into account in the analysis of the epidemic scales and concluded, by the way, as in many other works, that further improvement of the mathematical model is required, in particular, a more reliable and objective method of setting the model parameters is required. Therefore, most papers on this topic are devoted, for example, to [8; 9], on specific issues of epidemic modeling.

An analysis of recent publications shows that there is currently no universal mathematical model to describe the dynamics of the epidemic. In this paper, an attempt is also made to mathematically model the specific situation of the abstract epidemic with appropriate assumptions in the formation of initial conditions.

Research results. We construct a mathematical model of the epidemic of one of the individual cases that are common in life. Suppose that a population of people consisting of *N* persons is divided into three groups. The first includes individuals who are susceptible to a particular disease while still healthy. The number of such persons at a given time τ is denoted as *A(τ)*. The second group is people who are infected - they are sick themselves and are a source of disease spread. The number of such persons in the general population at time τ is denoted as *B(τ)*. And the third group are people who are healthy and immune to the disease. The number of such people at time ***τ*** is denoted as *C(τ)*. So:

$$A(\tau) + B(\tau) + C(\tau) = N. \quad (1)$$

Suppose that when the number of infected people exceeds a certain fixed number *B**, the rate of change in the number of people susceptible to the disease will be proportional to the number of the most susceptible persons. We also assume that the rate of change in the number of infected but recovering persons will be proportional to the number of infectious people. Of course, these assumptions somewhat simplify the real situation, but in the case they reflect the true situation. According to the first assumption, we will assume that when the number of infectious people is *B(τ)>B**, then they are capable of infecting susceptible people. This means that the isolation (up to some point in time) of infectious persons in quarantine is taken into account, or they are at a considerable distance from susceptible persons to the disease. Thus, we arrive at the differential equation:

$$\frac{dA}{d\tau} = \begin{cases} -\alpha A, \text{if } B(\tau) > B^* \\ 0, \text{if } B(\tau) \le B^* \end{cases} \quad (2)$$

It must be borne in mind that every person susceptible to the disease, after becoming ill, becomes infectious. Therefore, the rate of change in the number of infectious persons is the difference in unit time between those who have just become ill and those who are already recovering.

So,

$$\frac{dB}{d\tau} = \begin{cases} \alpha A - \beta B, \text{if } B(\tau) > B^* \\ -\beta B, \text{if } B(\tau) \le B^* \end{cases} \quad (3)$$

We call the constants α and β the morbidity and recovery coefficients, respectively.

The rate of change in the number of people recovering is given by the equation:

$$\frac{dC}{d\tau} = \beta B \quad (4)$$

To solve these equations uniquely, you must set the initial conditions. We assume that at time *τ=0*, there are no persons immune to the disease, that is, *C(0)=0* and *B(0)*. Also assume that the coefficients of morbidity and recovery are the same, that is *α=β*. We will look at this simple version of the situation. In real life, in most cases *α ≠ β*. We will cover this case in the next article.

Once these assumptions are made, there are two cases to consider.

Case 1. The number B (0) ≤B *

In this situation, with increasing time, individuals in the community will not be infected, because in this case $\frac{dA}{d\tau} = 0$ and according to equation (1) and the condition *C(0)*=0, for all *τ* is true equality:

$$A(\tau) = A(0) = N - B(0) \quad (5)$$

The case we are considering corresponds to the situation when most infectious persons are in isolation. In this case, from equation (3) we arrive at the differential equation:

$$\frac{dB}{d\tau} = -\alpha B \quad (6)$$

Hence $B(\tau) = B(0)\, exp(-\alpha\tau)$ and, respectively,

$$C(\tau) = N - A(\tau) - B(\tau) = B(0)[1 - \exp(-\alpha\tau)] \quad (7)$$

In Fig. 1 graphically shows the time change in the number of persons in each of the three groups.

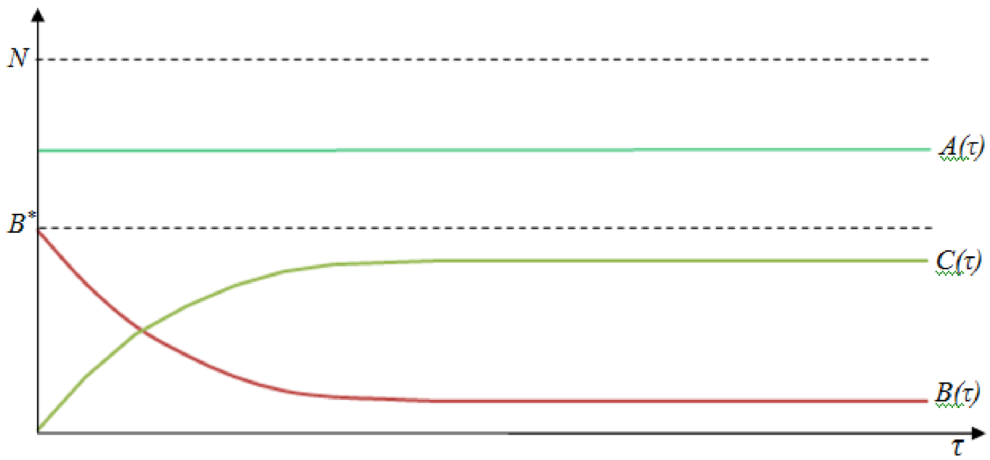

*Fig. 1 Dependence of change of number of persons in each of three groups at B(0)≤B**

Case 2. The number *B(0)>B**.

In this case, there must be a time interval *0≤τ<T*, for all values of *τ*, the inequality *B(τ)>B** holds, since the content of problem *B* must be a function of time *τ*. From which it follows that for all τ from the interval *[0, T]* the disease will spread to susceptible persons. Thus from equation (2) it follows that:

$$A(\tau) = A(0)\, exp(-\alpha\tau) \quad (8)$$

for *0 ≤ τ ≤ T*.

Substitute the value of *A(τ)* from (8) in equation

(3) and proceed to the differential equation:

$$\frac{dB}{d\tau} + \alpha C = \alpha A(0)\,exp(-\alpha\tau) \qquad (9)$$

If we multiply both parts of equation (9) by $exp(\alpha\tau)$, then this equation will take the form:

$$\frac{d}{d\tau}[B\ exp(\alpha\tau)] = \alpha A(0) \qquad (10)$$

After integration (10) we will have:

$$B\ exp(\alpha\tau) = \alpha A(0)\tau + K$$

and accordingly the set of all solutions of equation (9) is given by the relation:

$$B(\tau) = K\,exp(-\alpha\tau) + \alpha A(0)\tau\,exp(-\alpha\tau) \qquad (11)$$

If we take here $\tau=0$, we choose $K=B(0)$, and thus equation (11) takes the form:

$$B(\tau) = [B(0) + \alpha A(0)\tau]\,exp(-\alpha\tau) \qquad (12)$$

for $0 \le \tau \le T$.

In the future we need to find the true value of T and find the time point τ_{max} at which the number of infectious persons will be maximum.

Finding the time T is important because at this point the morbidity of susceptible persons stops. If we take $\tau=T$ in equation (12), then taking into account the previous conclusion, we can see that its right part takes the value B^*, that is:

$$B^* = [B(0) + \alpha A(0)T]\,exp(-\alpha T) \qquad (13)$$

But $A(T) = \lim_{\tau\to\infty} A(\tau) = A(\infty)$ is a number.

Susceptible persons who have escaped the disease and for whom the following equations are fulfilled:

$$A(T) = A(\infty) = S(0)\,exp(-\alpha T) \qquad (14)$$

From the last equation we can find T:

$$T = \frac{1}{\alpha} ln \frac{A(0)}{A(\infty)} \qquad (15)$$

Thus, if we can specify the number of persons $A(\infty)$, we can thus use equation (15) to predict the time of epidemic completion. After substituting T from (15) into equation (13) we obtain the equation:

$$B^* = \left[B(0) + A(0)\,ln\frac{A(0)}{A(\infty)}\right]\frac{A(\infty)}{A(0)}$$

or $$\frac{B^*}{A(\infty)} = \frac{B(0)}{A(0)} + ln\frac{A(0)}{A(\infty)}$$

which can be rewritten as:

$$\frac{B^*}{A(\infty)} + ln\,A(\infty) = \frac{B(0)}{A(0)} + ln\,A(0) \qquad (16)$$

Since B^* and all terms in the right-hand side of equation (16) are known, we can use it to determine $A(\infty)$.

To find the time τ_{max} at which the number of infected persons will be greatest, consider equation (12), from which you can go to equality:

$$\frac{dB}{d\tau} = [\alpha A(0) - \alpha B(0) - \alpha^2 A(0)\tau]\,exp(-\alpha\tau) = 0 \qquad (17)$$

From this equation we can find the time τ_{max} for which B reaches the maximum value:

$$\tau_{max} = \frac{1}{\alpha}\left[1 - \frac{B(0)}{A(0)}\right] \qquad (18)$$

Now substitute the value τ_{max} from the last equality in (12) and obtain:

$$B_{max} = A(0)exp\left\{-\left[1 - B(0)/A(0)\right]\right\} = A(\tau_{max}) \qquad (19)$$

The obtained equation shows, in particular, that at time τ_{max} the number of disease-prone individuals coincides with the number of infectious people.

When $\tau > T$, susceptible persons no longer become infectious and:

$$B(\tau) = B^* exp[-\alpha(\tau - T)] \qquad (20)$$

Figure 2 schematically shows the change in the number of persons in each of the three groups at $B(0)>B^*$.

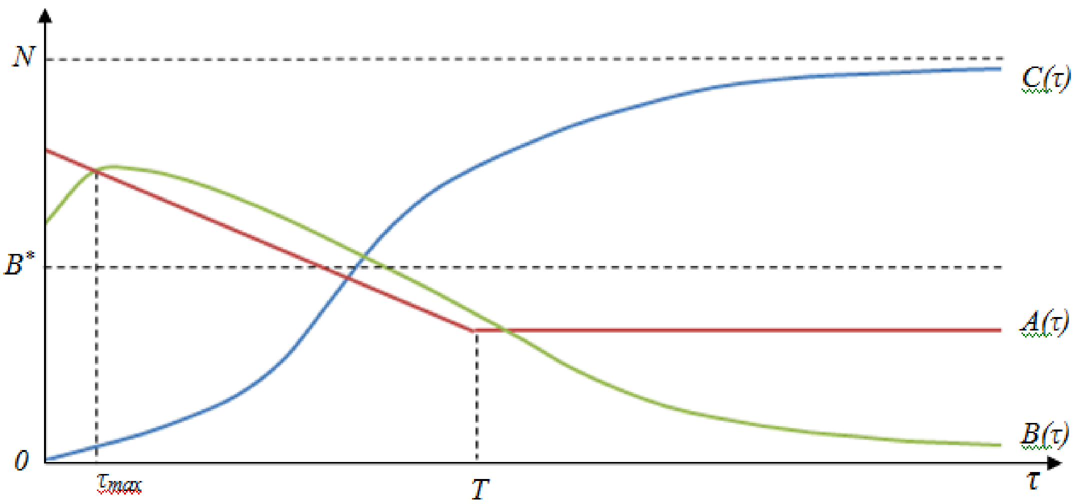

Fig. 2 Dependence of change of number of persons in each of three groups at B(0)>B.*

Conclusions. According to the results of the researches, a model of the epidemic development dynamics in the form of differential equations was constructed. An expression was obtained to determine the time at which the number of infected persons would be maximal. An equation was obtained to predict the timing of the epidemic. It depends on the ratio of people in the first and second groups. The graphical interpretation of the change in the number of persons of three population groups is given.

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К ВОПРОСУ О ПРОСТРАНСТВЕ И ВРЕМЕНИ

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TO THE QUESTION OF SPACE AND TIME

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АННОТАЦИЯ

Современный этап развития науки и техники предполагает разрешение проблем связанных, как минимум, с познанием организации информационной обработки, которая размещается в естественном интеллекте. А также насущной проблемой выступает создание средств передвижения со скоростями, которые не затягивают время пребывания человека космосе. Ведь оно может отрицательно сказываться на его здоровье и существенно ограничивать время познания. Без адекватной природе ориентации в таких понятиях, как пространство и время, научному сообществу разрешить, отмеченные выше проблемы затруднено. В статье исследуются современные подходы в создании системы пространства и времени и предложена попытка построить адекватную природе модель.

ABSTRACT

The current stage in the development of science and technology involves solving problems related, at least, to the knowledge of the organization of information processing, which is located in natural intelligence. And also an urgent problem is the creation of vehicles with speeds that do not delay the time a person spends in space. After all, it can adversely affect his health and significantly limit the time of knowledge. Without an orientation adequate to the nature in concepts such as space and time, it is difficult for the scientific community to resolve the problems noted above. The article explores modern approaches to creating a space and time system and suggests an attempt to build a model adequate to nature.

Ключевые слова: трехмерное пространство, время материальная точка, материальная линия, материальная площадь, мгновение, бесконечность

Keywords: three-dimensional space, time material point, material line, material area, instant, infinity

1. Введение

В конце 19-о и начале 20-о столетий в физике разразился серьезный кризис, и его разрешение, судя по всему, можно было бы осуществить аксиоматическим методом исследований, это когда на основе системы аксиом в алгебре доказываются любые леммы и теоремы. Иными словами, используя аналогичную систему, но уже аксиом в физике, именуемых постулатами, познать любое явление в природе. По-видимому, с этой целью в 1900 году, среди 23 проблем, Д. Гильберт сформулировал шестую проблему, в разрешении которой предполагалось «математически обосновать систему аксиом физики». Как потом выяснилось, такую фиксированную систему аксиом естествознания построить невозможно [1], а вот «открытую» систему, которая, по мере познания, пополняется новыми посту-

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