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Research Paper

Angular oscillation model to predict the performance of a vibratory ball mill for the fine grinding of grain

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The development of a mathematical model of a vibratory mill made it possible to determine the regularities for the complex movement of its operating mechanisms and obtain a graphic interpretation for the kinematic, power and energy characteristics of the vibratory system. On the basis of theoretical pre-requisites, a series of experimental studies were conducted that provided an opportunity to obtain the amplitude-frequency, velocity and energy characteristics of the machine under investigation and to determine their impact upon the kinetics of the fine grinding process of the grain. As a result, rational operating parameters of the examined machine were established with minimal energy input.

1. Introduction

Grain refinement is the oldest technology used by humans to produce foodstuffs. The main technological task of grinding grain is to obtain a homogeneous mixture with a desired degree of grinding of the ingredients (Butkovsky, 1990; Linch, 2005). The refining process at the mill allows components such as the parts of the wheat grain to be separated: the endosperm containing mostly starch and proteins, the germ composed mostly of lipids and proteins and the bran containing mainly dietary fibre (Liu et al., 2015; Marquart, Jacobs, McIntosh, Reicks, & Poutanen, 2007). At present, a wide variety of mills and crushers are used for the production of flour for food and animal fodder (Cho, Kwon, Kim, & Mun, 2013; Savinyh, Nechaev, Nechaeva, & Ivanovs, 2016; Sysuev, Savinyh, Aleshkin, & Ivanovs, 2016). As example,

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the production of whole grain flour is mainly affected by the milling process and the predominant techniques for grinding whole grain flours are stone mills, roller mills, ultra-fine mills and hammer mills (Kent & Evers, 1994; Kihlberg, Johansson, Kohler, & Risvik, 2004). Research in the intensification of grain refinement is reflected in many works (Bulgakov, Holovach, Bandura, & Ivanovs, 2017; Rajamani, Mishra, Venugopal, & Datta, 2000; Rosenkranz, Breitung-Faes, & Kwade, 2011; Sysuev, Ivanovs, Savinyh, & Kazakov, 2015); however, the operation of a vibratory mill for fine grinding grain lacks sufficient research. As a result of using vibration, there is a significant decrease in the coefficient of internal friction and an increase in the specific surface area of the material processed when interacting with the technical filler essentially creating gradients in the rates of internal deformations in the micro- and macro-volumes of the product.

When fine grinding, vibration makes it possible to substantially increase the shock-erasing effect due to the possible variations in its strength and frictional components and to increase the rate destruction of the grain under the impact of cyclic loads as a result of dynamic interaction between particles (Doumanidis et al., 2016; Gonzalez, 1995; Janovich, Polevoda, & Nurmetov, 2016; Mori, Mio, Kano, & Saito, 2004). The main shortcomings of the traditional machines for producing fine and highly disperse material relates to the specific energy input required for processing the raw materials, the low performance characteristics of the operating mechanisms due to the active abrasion of their operating surfaces, and reduction in technical efficiency as a result of adhesion of products with increased humidity (Manetto, Cerruto, Pascuzzi, & Santoro, 2017; Kaletnik, 2006; Nasir, 2005; Yaroshevich, 2011).

In order to eliminate the above-mentioned shortcomings and increase the operational and technical performance of grinding machines, it is proposed to investigate using complex vibrational impacts.

The scientific purpose of this study was to develop a mathematical model of a vibratory mill and to carry out experiments to substantiate the technical parameters of its operation under the conditions required for intensified grain refinement, and to improve the operational and technical indicators on the basis of using complex vibro-mechanical impacts.

2. Materials and methods

An experimental vibratory mill using angular oscillations was used for this study (Franchuk, 1970, pp. 193–197; Janovich, 2015) but theoretical investigations to understand the process were carried out here. The energy consumption of the developed machine, its amplitude-frequency, kinetic and its operational parameters were estimated on the basis of experimental measurements, but analysis of the kinematic characteristics of the vibrator (vibration acceleration, vibration velocity and vibration displacement) was conducted using software developed for analysing grinding.

2.1. *The measure chain*

In order to determine the amplitude-frequency characteristics, an accelerometer analyser was developed, based on the 3-axis accelerometer LIS3DH (STMicroelectronics, USA), whose main features were: dynamically user-selectable full scales of ±2 g/±4 g/±8 g/±16 g, capability of measuring accelerations with output data rates from 1 Hz to 5.3 kHz.

The principle of operation of this device is the following: after connecting the sensor to the surface of the grinding chamber of the mill, the drive mechanism of the machine is switched on, due to which the resulting vibrations of the grinding chamber initiate inclusion of the integrated accelerometer that starts registration of the amplitude-frequency characteristics of the machine under study. To record the rotation speed of the drive shaft, the UNI-T UT372 (Uni-Trend Technology Limited, China), wireless tachometer was used, whose main technical features were measurements from 10 to 99,999 RPM with an accuracy 0.04% +/− 2dgt. The control and selection of the rotational speed of the motor shaft was carried out with the help of the autotransformer AOSN-20-220-75 (Zapadpribor, Lviv, Ukraine), which is intended for operation with alternating current. To determine the energy characteristics of the machine studied, an electronic wattmeter EMF-1 (Tequipment.net, Long Branch, NJ, USA) was used.

The dispersion ability of the material was determined using sieve analysis using a laboratory sieve analyser A-20 (LabTime, Moscow, Russia), whose main technical features were: diameter of screen 200 mm, number of screen 4–10, amplitude of oscillations − 2 to 4, frequency of oscillation 1500 rad s^{-1}, granularity 0.008–5 mm. It was used as standard screen machine grading all kinds powder material particle. In order to determine the relative humidity of the material, a Wile 55 moisture meter (Farmcomp Oy, Tuusula, Finland) was used to measure the moisture content of various types of grain and seeds, the data being stored in the memory of the instrument. The Wile 55 m can measure a wide range of moisture, grain 8–35% and oil seeds 5–25%. Wile 55 is the basic tool for precise quick measurement of grain moisture. It has automatic temperature compensation that utilises the internal temperature sensors for both grain and the temperature of the device itself. To determine the specific surface area of the treated material during the vibratory grinding process with the particle size being in the range 25–1000 μm, a PSH-9 (Company Hranat, Moscow, Russia) that had the following main features: range of specific surface area of the material 0.03–5 m^2 g^{-1}, time of one analysis 3.5 min, accuracy 0.5%. This instrument measures the permeability of a layer of material through which air is leaking under pressure close to the atmospheric one.

To analyse the quality of the crushed raw material, a laboratory sampler was applied for sampling by the point selection method. To control the supply of material, a mobile vibratory dispenser PG-2 (Company Vybrotekhnyk, Russia) was used, whose main technical features were: size of the source material 0.05–5 mm, productivity 400 kg h^{-1}, amplitude of oscillation of the tray 2 mm, bunker volume 9 dm^3.

Thus, the presented structural design for carrying out laboratory and production tests, shown in the block diagram of Fig. 1, allows us to determine precisely enough the main technological parameters of the angular oscillations of the developed mill and to evaluate the quality of the material obtained within the range of experimental studies.

2.2. *The developed vibratory mill*

The principle of operation of the developed vibratory mill (Janovich, 2014) is the following: when the electric motor (1 in Fig. 2) is switched on, the rotational torque is transmitted through the elastic clutch (2 in Fig. 2) to the drive shaft (3 in Fig. 2) with unbalanced masses (4 in Fig. 2) arranged on it, rotation of which ensures generation of a combined force and moment imbalance of the oppositely arranged (on the periphery from the two sides) in relation to the central axis (5 in Fig. 2) grinding chambers (6 in Fig. 2) filled to 75% with the grinding elements in the form of steel balls. These grinding chambers have the form of cylinders, the volume of which is 10^{-2} m^3 each. The inclination angle of the grinding chamber in the horizontal plane is 15°. As a technical filler, steel balls with a diameter of 8 mm were chosen. The grinding chambers, in their turn, are interconnected by traverses (crosspieces) (7 in Fig. 2). This design solution of the machine allows to provide angular oscillations, ensuring a significant dynamic state of the technological filler (promoting active collision of the grinding elements), and, as a consequence, it provides a possibility to increase the grinding efficiency of the raw material being processed continuously and entering through the feed pipes (8 in Fig. 2) from the hopper (9 in Fig. 2). Grinding takes place due to cyclic loads created by the crushing elements and their dynamic interaction with each other. When the particle size of the material to be crushed is reduced under the impact of the centrifugal forces and reversal loads, it is transported along the grinding chamber to the separating section where particles, smaller than the separation holes of the sieve, are discharged from the mill through the pipe (10 in Fig. 2); the particles of the material that have a larger size – are crushed due to the active impact of crushing elements.

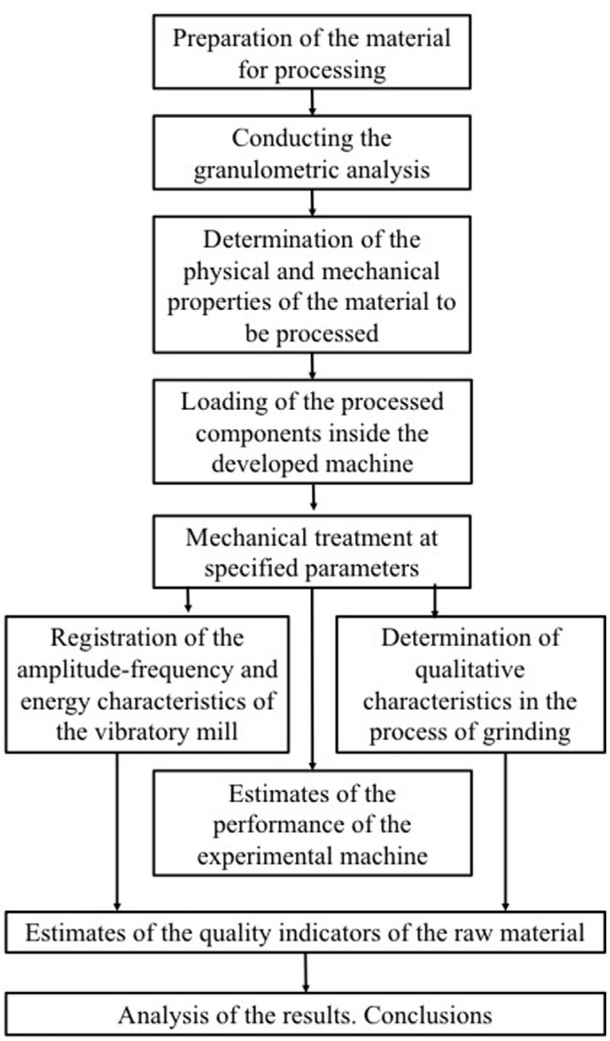

Fig. 1 – Block diagram concerning the preparation and conduct of experimental investigations.

An electric motor with a power of 0.75 kW and an operating rotation speed of 1500 min^{-1} was used. Preliminary studies of the experimental mill showed that it had a throughput of 220 kg h^{-1}. This result was obtained grinding maize, peas, rye and wheat with moisture content in the range 8–11%, replicating the measurement 12 times for each condition. A total of 31 tests were performed with a 5-fold variation in the technical parameters of the developed machine (Janovich et al., 2016). To study the control of the process for the fine grinding of bulk raw materials, it was proposed to determine the internal structure of the dynamical system under investigation and to find linear and nonlinear effects, and on this basis to formulate a mathematical model of the vibratory system (Dreizler & Ludde, 2010; John & Stephens, 1984, pp. 213–224; Maitra & Prasad, 1985).

2.3. *The mathematical model*

In order to create a mathematical model of angular oscillations of the vibratory mill (based on the D'Alembert-Euler principle), an equivalent scheme was designed, characterised by five degrees of freedom (Fig. 3). Accordingly, the differential equations of the movement of the operating mechanism of the machine are described by five independent generalised coordinates, where ρ is the radial coordinate of positioning the grinding chamber relative to axes Oz, Ox, m; φ is the displacement angle of the grinding chamber with respect to the vertical plane, rad; α is the arrangement angle of the vibratory drive relative to the container, rad; φ_1 is the displacement angle of the vibrator drive depending on the change of the position of the grinding chamber, rad; φ_3 is the angle of rotation of the unbalance relative to its axis of rotation, rad; O_{xz} is the fixed coordinate system; O_1 is the centre of the container mass; c_x, c_z are the spring stiffness in the direction of the corresponding axis, N m^{-1}; F_c is the force of elasticity, N; and F_k is the centrifugal force, N.

Considering the characteristics of the angular movement of the grinding chamber of the machine, the mathematical model of the dynamic system was formed in a polar coordinate system. The prerequisite for theoretical investigations was a search of operational and design parameters of the

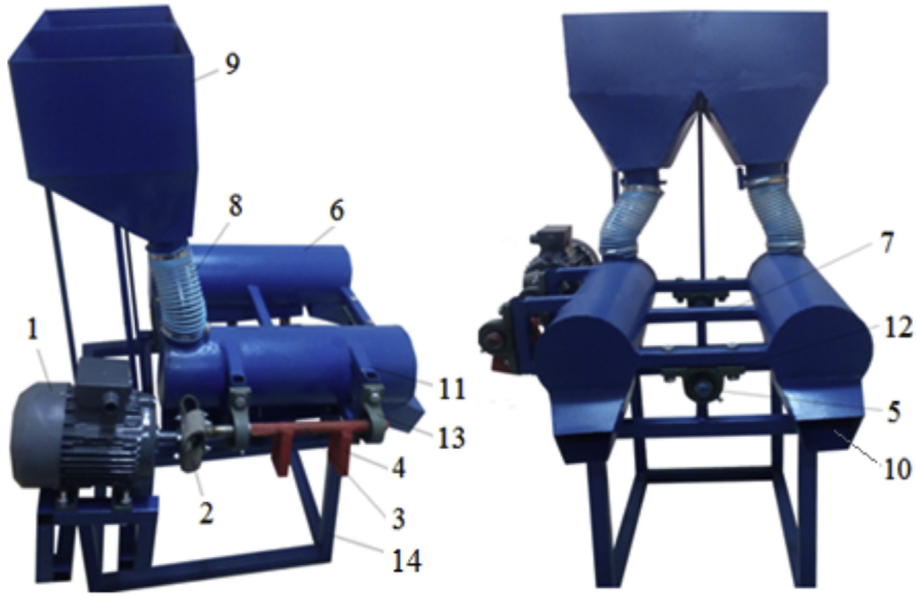

Fig. 2 – General view of the vibratory mill with angular oscillations: 1 – electric motor; 2 – flexible coupling; 3 – shaft; 4 – unbalances; 5 – central axis; 6 – grinding chamber; 7 – traverses (crosspieces); 8 – feed pipes; 9 – hopper; 10 – discharge pipe; 11 – bearing supports; 12, 13 – bearing units assemblies; 14 – frame.

developed machine in which the vertical component of the vibrations of its grinding chambers will acquire its maximum value. It is possible to find out the relationship between the design and technological parameters using the initial Lagrange equations of the second kind.

The kinetic energy of the grinding chamber can be described by

$$T_c = \frac{1}{2}\left(m_c V_{O_1}^2 + J_c \dot{\varphi}_1^2\right) \tag{1}$$

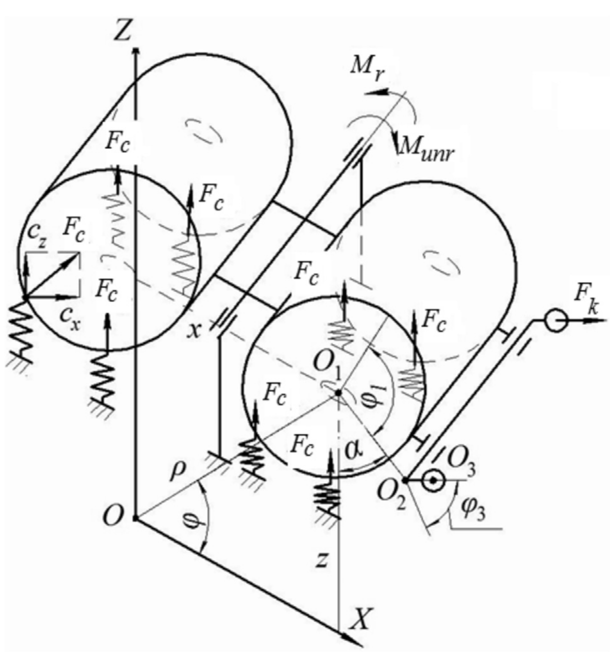

Fig. 3 – Equivalent scheme of the vibratory mill with angular oscillations.

where m_c is the mass of the container, kg; J_c is the inertia moment of the container, kg m^2; $\dot{\varphi}_1$is the angular velocity of the container, rad s^{-1}; V_{O_1} is the velocity of point O_1 in the polar coordinate system described by

$$V_{O_1}^2 = \dot{\rho}^2 + (\rho \cdot \dot{\varphi})^2 \tag{2}$$

In its turn, the kinetic energy of the unbalance will be equal to:

$$T_{unb} = \frac{1}{2}\left(m_{unb} V_{O_3}^2 + J_{unb} \dot{\varphi}_3^2\right) \tag{3}$$

where m_{unb} is the mass of the unbalance, kg; J_{unb} is the inertia moment of the unbalance, kg m^2; $\dot{\varphi}_3$is the angular velocity of unbalance, rad s^{-1}; $V_{O_3}$is the velocity of the centre of masses of unbalance, that is determined using the polar coordinate system, where the pole to the system is located at this point. The speed $V_{O_3}$is then determined according to the following vector equation:

$$\overline{V}_{O_3} = \overline{V}_{O_1} + \overline{V}_{O_2O_1} + \overline{V}_{O_3O_2} \tag{4}$$

where $\overline{V}_{O_1}$ is the vector velocity of point O_1 in the polar coordinate system; $\overline{V}_{O_2O_1}$ is the vector velocity of point O_2 relatively point O_1; and $\overline{V}_{O_3O_2}$ is the vector velocity of point O_3 relatively point O_2;

In this case, the expression for the kinetic energy of the unbalance has the form:

$$\begin{aligned} T = {} & \frac{1}{2}m_c\left(\dot{\rho}^2 + \rho^2\dot{\varphi}^2\right) + \frac{1}{2}J_c\dot{\varphi}_1^2 + \frac{1}{2}m_{unb}\left[\dot{\rho}^2 + \rho^2\dot{\varphi}^2 + \dot{\varphi}_1^2 l_2^2 + \dot{\varphi}_3^2 l_3^2\right. \\ & + 2\dot{\rho}\dot{\varphi}_1 l_2 \cos(\varphi - \varphi_1 - \alpha) + 2\dot{\rho}\dot{\varphi}_3 l_3 \sin(\varphi - \varphi_1 - \varphi_3) \\ & + 2\rho\dot{\varphi}\dot{\varphi}_1 l_2 \sin(\alpha + \varphi_1 - \varphi) + 2\rho\dot{\varphi}\dot{\varphi}_3 l_3 \cos(\varphi - \varphi_1 - \varphi_3) \\ & \left. + 2\dot{\varphi}_1 l_2 \dot{\varphi}_3 l_3 \sin(\alpha - \varphi_3)\right] + \frac{1}{2}J_{unb}\dot{\varphi}_3^2. \end{aligned} \tag{5}$$

As a result of the partial derivatives found for the corresponding generalised coordinates, the generalised forces of the system and their functional transformations, a general form of the Lagrange equations of the second kind was obtained that fully determined the movement regularity of the grinding chamber of the experimental mill. In particular, the dependence of the radial coordinate of positioning ρ the grinding chamber relative to axes O_z, O_x was determined, by respectively, Eqs (6) and (7):

$$(m_c + m_{unb})\ddot{\rho}\varphi_1 - m_c\rho\dot{\varphi}^2 + m_{unb}\Big[-\rho\dot{\varphi}^2 + \ddot{\varphi}_1 l_2 \cos(\varphi - \varphi_1 - \alpha) + (\ddot{\varphi}_1 + \ddot{\varphi}_3)l_3 \sin(\varphi - \varphi_1 - \varphi_3) + \dot{\varphi}_1^2 l_2 \sin(\varphi - \varphi_1 - \alpha) - (\dot{\varphi}_1 + \dot{\varphi}_3)^2 \times l_3 \cos(\varphi - \varphi_1 - \varphi_3)\Big] = -(c_x \cos^2\varphi + c_z \sin^2\varphi)\rho; \tag{6}$$

$$(m_c + m_{unb})\rho^2\ddot{\varphi} + 2(m_c + m_{unb})\rho\dot{\rho}\dot{\varphi} + m_{unb}\Big[\rho\ddot{\varphi}_1 l_2 \sin(\alpha + \varphi_1 - \varphi) + \rho\dot{\varphi}_1^2 l_2 \cos(\alpha + \varphi_1 - \varphi) + \rho(\ddot{\varphi}_1 + \ddot{\varphi}_3)l_3 \cos(\varphi - \varphi_1 - \varphi_3) + \rho(\dot{\varphi}_1 + \dot{\varphi}_3)^2 l_3 \sin(\varphi - \varphi_1 - \varphi_3)\Big] = (c_x - c_z)\rho^2 \sin\varphi \cos\varphi; \tag{7}$$

where M_r is the rotation torque on the drive shaft, N m; M_{unr} is the moment of the support on the drive shaft, N m; ω, $\omega_{\max}$, ω_w is the respective initial, maximal and operating angular velocity of the drive shaft, rad s^{-1}.

Assuming that Mr is a constant, considering and that $\omega = \omega_3$, ωw is constant, ωr is constant. Solutions were sort such that:

$$\rho(t) = \rho_0 + \rho_1(t), \tag{11}$$

where ρ_0 is the initial value ρ, m; $\rho_1 = \varphi_3(t) = \omega t$ is the value of ρ as the function of time t, m; $\varphi(t) = \omega t + \varphi_0(t)$ is the value of angle φ, which is due to the rotational movement of the unbalance.

As a result of solving and simplifying the obtained equations, based on the trigonometric connection of the polar and Cartesian coordinate systems, the obtained regularity of the movement of the grinding chamber of the mill will assume the form:

$$\left.\begin{aligned} z(t) &= -\frac{m_{unb} l_3 \sin\omega t}{m_c + m_{unb}} + l_2 \sin\left(\frac{3\pi}{2} + \frac{(m_{unb} m_c l_2 l_3 \cos(\omega t - \alpha))\cos(\omega t - \alpha)}{\left(J_c + J_{unb} + m_{unb}\left(l_2^2 + l_3^2\right)\right)(m_c + m_{unb})}\right), \\ x(t) &= -\frac{m_{unb} l_3 \sin\omega t}{m_c + m_{unb}} - l_2 \cos\left(-\frac{(m_{unb} m_c l_2 l_3 \cos(\omega t - \alpha))\cos(\omega t - \alpha)}{\left(J_c + J_{unb} + m_{unb}\left(l_2^2 + l_3^2\right)\right)(m_c + m_{unb})}\right). \end{aligned}\right\} \tag{12}$$

Whereas, the regularity of the displacement angle of the grinding chamber φ with respect to the vertical plane is determined by means of Eq. (8):

$$J_c\ddot{\varphi}_1 + J_{unb}(\ddot{\varphi}_1 + \ddot{\varphi}_3) + m_{unb}\Big[\ddot{\varphi}_1 l_2^2 + (\ddot{\varphi}_1 + \ddot{\varphi}_3)l_3^2 + \ddot{\rho} l_2 \cos(\varphi - \varphi_1 - \alpha) + \ddot{\rho} l_3 \sin(\varphi - \varphi_1 - \varphi_3) + \rho\ddot{\varphi} l_2 \sin(\alpha + \varphi_1 - \varphi) + \rho\ddot{\varphi} l_3 \cos(\varphi - \varphi_1 - \varphi_3) + (2\ddot{\varphi}_1 + \ddot{\varphi}_3)l_2 l_3 \sin(\alpha - \varphi_3) - 2\dot{\rho}\dot{\varphi} l_2 \sin(\varphi - \varphi_1 - \alpha) + 2\dot{\rho}\dot{\varphi} l_3 \times \cos(\varphi - \varphi_1 - \varphi_3) - \rho\dot{\varphi}^2 l_2 \cos(\alpha + \varphi_1 - \varphi) - \rho\dot{\varphi}^2 l_3 \sin(\varphi - \varphi_1 - \varphi_3) - (2\dot{\varphi}_1 + \dot{\varphi}_3)\dot{\varphi} l_2 l_3 \cos(\alpha - \varphi_3)\Big] = -c_z\varphi_1 - m_{unb}[l_2 \sin(\alpha + \varphi_1) + l_3 \cos(\varphi_1 + \varphi_3)]; \tag{8}$$

In turn, the displacement angle of the vibrator drive φ_1 depending on the change of the position of the grinding chamber is determined by Eq. (9):

$$J_{unb}(\ddot{\varphi}_1 + \ddot{\varphi}_3) + m_c\Big[(\ddot{\varphi}_1 + \ddot{\varphi}_3)l_3^2 + \ddot{\rho} l_3 \sin(\varphi - \varphi_1 - \varphi_3) + \rho\ddot{\varphi} l_3 \cos(\varphi - \varphi_1 - \varphi_3) + \ddot{\varphi}_1 l_2 l_3 \sin(\alpha - \varphi_3) + 2\dot{\rho}\dot{\varphi} l_3 \cos(\varphi - \varphi_1 - \varphi_3) + \dot{\varphi}_1^2 l_2 l_3 \cos(\alpha - \varphi_3) - \rho\dot{\varphi}^2 \times l_3 \sin(\varphi - \varphi_1 - \varphi_3)\Big] = M_{unr} - m_{unb}\dot{\varphi}_3^2 l_2 l_3 \cos(\varphi_1 + \varphi_3); \tag{9}$$

$$M_{unr} = \frac{2M_r(\omega_w - \omega_{\max})(\omega_w - \omega)}{(\omega_w - \omega)^2 + (\omega - \omega_{\max})^2}, \tag{10}$$

2.4. *Data analysis*

The processing of the experimental data was carried out using a multifactorial experiment and variational analysis of the factor space using Microsoft Excel, Statistic 10.0, FluidLab software environments. To assess the impact of the technical parameters of the developed vibrator on the efficiency of the grinding process the condition was that energy input is minimised. The statistical technique of rotatable central compositional planning of the multifactorial experiment was applied (Protasov, 2005).

Among the basic criteria for the evaluation of the grinding process of a bulk raw material, the energy consumption of the vibrator N, kW h and the specific surface area of the initial raw material S, m^2 g^{-1}, were characterised by the impact of four most significant factors that determine the kinetics of the particular treatment; the vibration acceleration factor a ($a = A \cdot \omega^2$), m·s^{-2}, which is a complex indicator of the dynamic state of the vibratory system and takes into account the change in the amplitude of the oscillations A of the grinding chamber of the machine and the angular velocity of the drive shaft of the unbalances ω; the diameter of the balls d, mm, and the degree of loading of the grinding chambers E, % with the technical filler.

The ranges of the factors of functions was chosen in such a way that any of their aggregates, as envisaged by the experimental design, could be embodied in these intervals and did not lead to contradictions.

3. Results and discussion

Figure 4 reflects the path of the movement of the grinding chamber of the machine depending on the change in the displacement angle of the vibrator drive by every 45°. The analysis of the obtained dependencies (12) in the FluidLab (Festo Didactic GmbH & Co., Denkendorf, Germany) software environment made it possible to establish that the maximum dynamic state of the system is provided at a 290° angle of the vibratory drive and the angular velocity of the drive shaft 120 rad s^{-1}. With these parameters the ratio $A = A_z/A_x$ of the components of the amplitude of the oscillations was 2.5 times (Fig. 5c). In this case, the displacement amplitude of the grinding container along axis O_z was $A_z = 5.4$ mm (Fig. 5b) and along the axis O_x was $A_x = 2.2$ mm (Fig. 5a).

It should be noted that the received graphic dependences of the amplitude of oscillations from the arrangement angle of the vibration drive, have a symmetric distribution relative to the x-axis. The point of their symmetry is 180°.

To verify the analytical results obtained from the theory, a number of experimental studies were carried out on the efficiency of fine grinding in the developed machine. It was established that the total amplitude of oscillations A at $\omega = 44$ rad s^{-1} without a technical load ($E = 0\%$) and at $E = 50\%$ and $E = 75\%$ reaches 3.8, whereas in the resonance mode of the investigated vibratory system in the range $\omega = 50–57$ rad s^{-1} the value of the required parameter without a technical filler ($E = 0\%$) was $A = 4.5$; at $E = 50\%$ $A = 4.2$; $E = 75\%$ $A = 4.0$. After that curve A in the range $\omega = 63–110$ rad s^{-1} stabilised by 3.7, regardless of the degree of loading of the technical filler (Fig. 6).

The data obtained made it possible to evaluate the efficiency of the developed design in the context of minimising the dissipative properties of the vibratory system; the criterion for estimating being the change in magnitude of the oscillation amplitude depending on the load of the grinding chamber of the machine with the technical filler. This trend was due to the presence of a peripherally located drive shaft with the oppositely placed grinding chambers of the mill, the forced force of which considerably exceeds the inertial characteristics of the technical filler. The machine was characterised by angular oscillations of the container relative to the centrally located axis. Therefore, the obtained data, with the dominance of the vertical component, did not contradict the concept of the vibrator which was to ensure a dynamic state of processing material (grain). The analysis of the dependence of the energy consumption of the mill upon its angular velocity and its loading with the technical filler can be seen in Fig. 7 where the degree of filling has no significant effect on the energy input. Thus, in the absence of a technical filler ($E = 0\%$) $N = 0.65$ kW h; at $E = 50\%$ $N = 0.68$ kW h; at $E = 75\%$ $N = 0.75$ kW h. On the basis of the obtained results one can conclude that introduction of the peripheral placement of the vibrator drive is one of the key factors for minimising energy

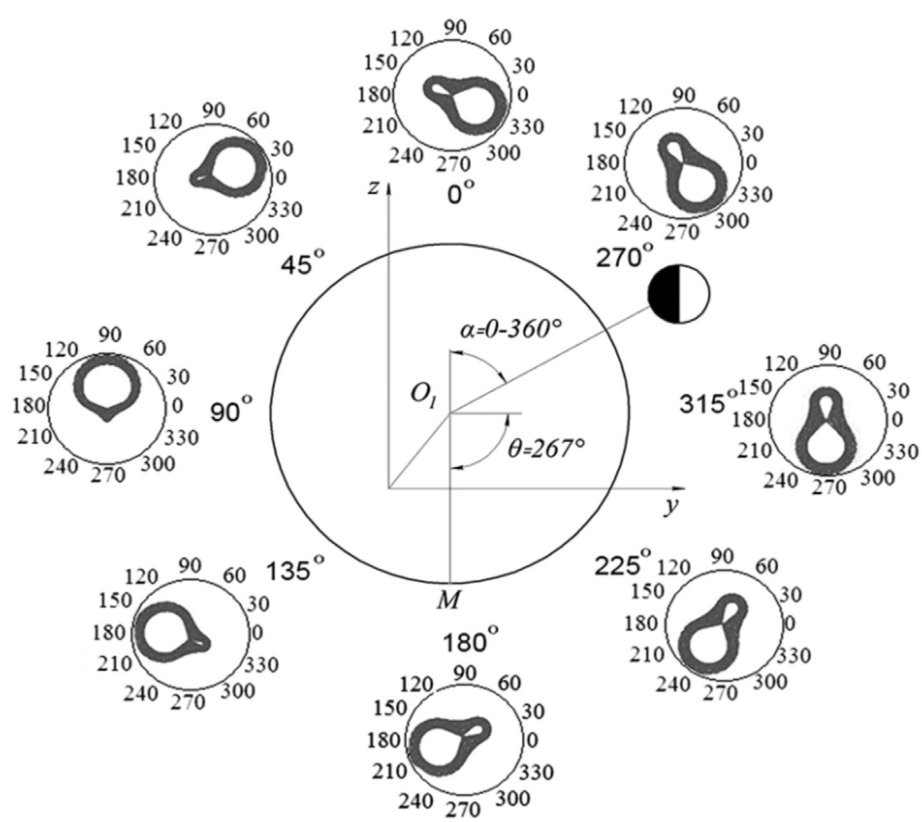

Fig. 4 – Kinetic characteristics of the grinding chamber of the mill, depending on the angle of placement of the vibratory drive α

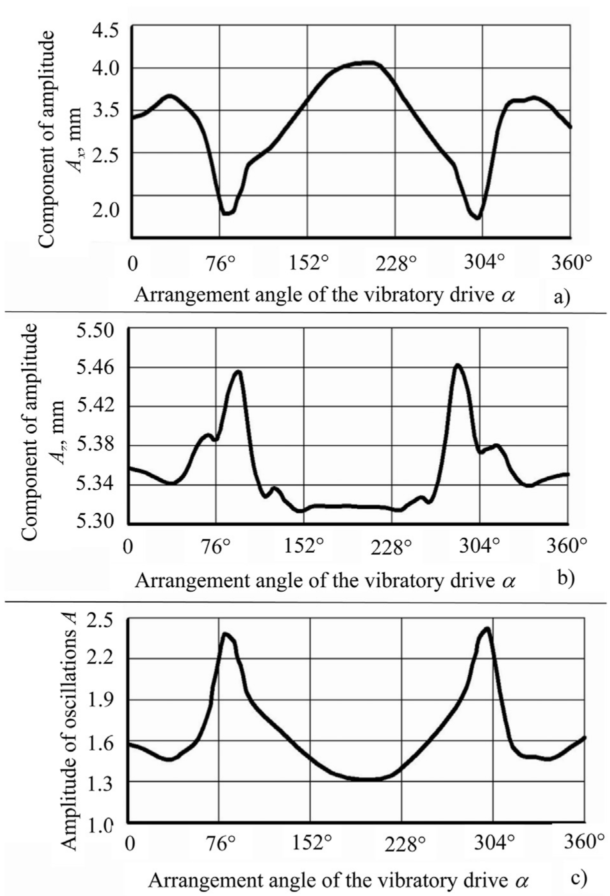

Fig. 5 – Dependence of the amplitude of oscillations A on the arrangement angle of the vibrator drive α: a) *Ax*, horizontal component of the oscillation amplitude; b) *Az*, vertical component of the oscillation amplitude; c) A, ratio of the aforesaid components of the amplitude.

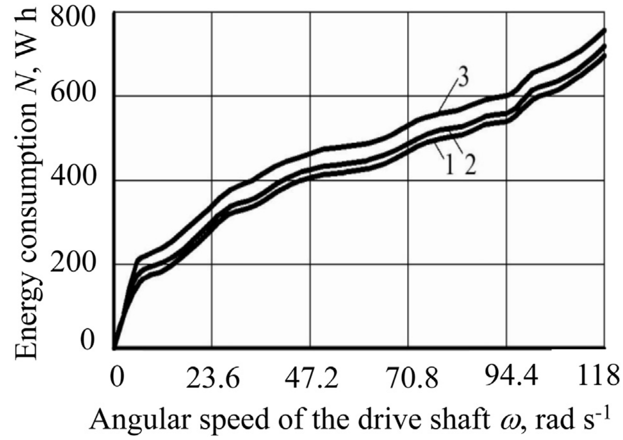

Fig. 7 – Dependence of energy consumption N of the mill with angular oscillations upon the angular velocity ω and loading of the technical filler E: 1) without a technical filler, E = 0%; 2) at E = 50%; 3) at E = 75%.

consumption in vibratory mills while maintaining a significant potential for bulk processing. In order to estimate the quality indicators of the vibratory grinding process of a bulk material processed in the mill, a number of experiments were carried out to change the dispersion ability of the treated raw materials, caused by the force of the technical filler (peas, oats and wheat grains with moisture content of 8–11% were used in the experiments). Evaluation of the grinding efficiency was based on the determination of the specific area S, m^2, the particle size distribution ΔS, μm, and the proper specific area of the material particles, which characterises the ratio of their geometric sizes before and after the treatment. When analysing the change in the specific area of the initial fraction of the wheat under the condition that a vibratory mill with angular vibrations (Fig. 8) is used, it was evident that increase in S that depended on the angular velocity of the drive shaft

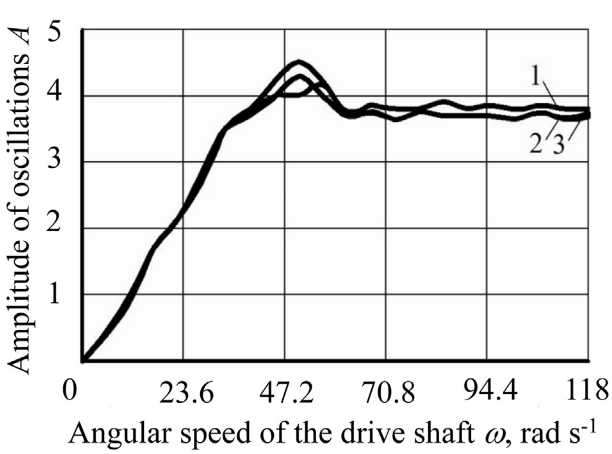

Fig. 6 – Total amplitude of oscillations A as function of the angular velocity ω of the drive shaft and the loading of the technical filler E: 1) without a technological filler, E = 0%; 2) at E = 50%; 3) at E = 75%.

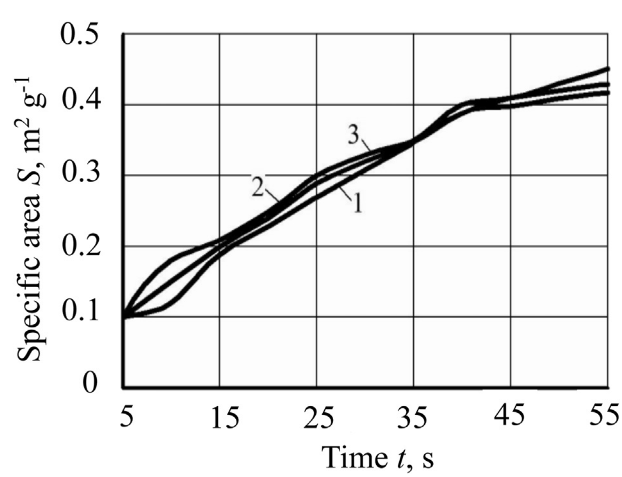

Fig. 8 – Dependence of the specific area of the wheat particles upon the angular velocity of the drive shaft ω and time t being in the grinding chamber of the mill: 1) ω = 90 rad s-1; 2) ω = 100 rad s-1; 3) ω = 110 rad s-1.

did not differ significantly, and at $\omega = 110$ rad s^{-1}, t = 55 s it was 0.43–0.45 m^2 g^{-1}. This trend was caused by a specific form of vibration, which considerably intensifies the process of active abrasion of the particles of the material processed.

With the increase in the specific area of the material, the proper area of the particles of the processed material decreased and, as a consequence, the time of treatment depended on the physical–mechanical parameters of the incoming raw material. On the basis of the obtained data about the kinetics of the pea, maize and wheat grinding process at $\omega = 110$ rad s^{-1} (Fig. 9), one can conclude that the approximate time of their processing will be t = 45 s at the particle size for the grinding of peas ΔS = 200–240 μm, for grinding of maize ΔS = 100–125 μm, for grinding of wheat ΔS = 50–100 μm.

Along with the obtained kinetic curves of the process under investigation, depending on the amplitude-frequency characteristics of the developed machine, there was a series of dependencies of the influence of the geometric parameters of the technical filler (balls) d upon the value of the specific area S of the crushed material (Fig. 10).

Based on the obtained data, one can conclude that the use of grinding balls with a diameter d = 10–18 mm is most expedient for coarse grinding S = 0.3 m^2 g^{-1}, but, when the degree of grinding was increased to S = 0.45 m^2 g^{-1}, the most suitable balls had a diameter d = 3–9 mm, which promoted significant increase in the number of their collisions and, as a consequence, lead to more active processing of the raw material. Analysis of the dependence of the specific area of the initial material upon the degree of loading of the grinding chamber with the technical filler testified that the highest value of S = 0.43–0.45 m^2 g^{-1} was observed at E = 60–75%. However, the filling percentage of the grinding chamber with the technical filler was always connected with increased energy consumption; therefore the choice of optimal operating conditions should be made by means of a compromise between the specific area of the initial fraction and the efficiency of the machine.

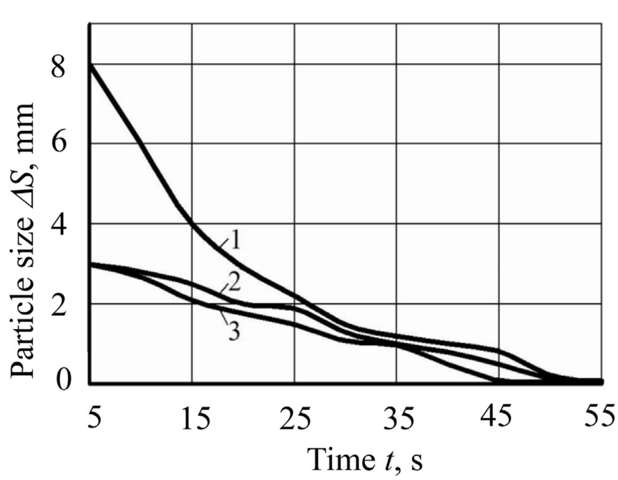

Fig. 9 – Dependence of the particle size of the processed material upon the time of its being in the grinding chamber at ω = 110 rad s-1: 1) grinding of peas; 2) grinding of maize; 3) grinding of wheat.

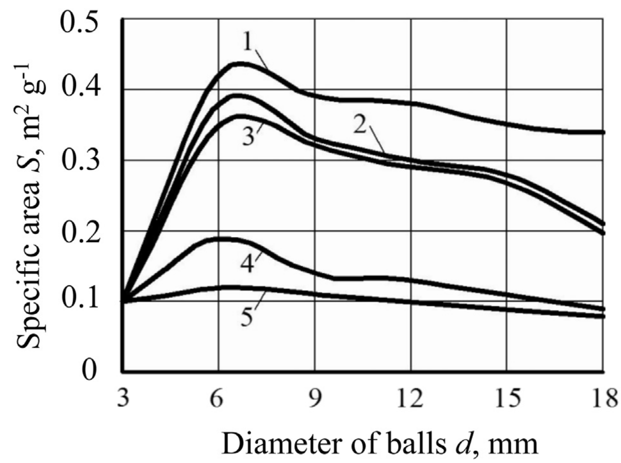

Fig. 10 – Dependence of the specific area of the wheat grain processed in the mill of angular vibrations depending on time t and diameter d of the crushing balls: 1) 50 s; 2) 40 s; 3) 30 s; 4) 20 s; 5) 10 s.

In order to evaluate the efficiency of the developed machine, a series of experiments were carried out to study the productivity of the developed angular oscillation mill, depending on the kinematic characteristics of the drive shaft and the diameter of the sieve holes d_s, under the condition of previously established operational and design parameters of the developed machine (Fig. 11). When analysing the dependence of the productivity of the vibratory mill with angular oscillations with a varying diameter d_s, it is worthwhile noting the narrow range of its values P = 100–110 kg h^{-1} at ω = 100–105 rad s^{-1}. However, a significant increase in the productivity P was observed by increasing the angular velocity of the drive shaft ω = 110–125 rad s^{-1}, which increased the movement of the processed material within the grinding chamber.

Thus, the maximum value of productivity in the operating mode of the developed mill ω = 110 rad s^{-1} was: for d_s = 0.5 mm P = 150 kg h^{-1}, for d_s = 1 mm P = 160 kg h^{-1}, and for d_s = 2 mm P = 168 kg h^{-1}. Evaluation of the investigated

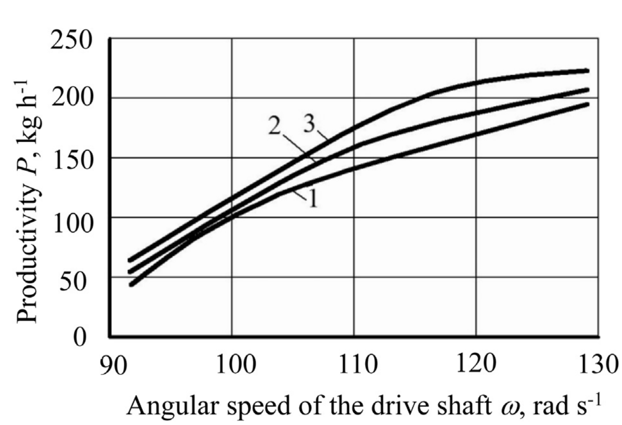

Fig. 11 – Dependence of the output of the angular oscillation mill upon the angular velocity of the drive shaft ω and the diameter of the screen holes ds: 1) ds = 0.5 mm; 2) ds = 1.0 mm; 3) ds = 2.0 mm.

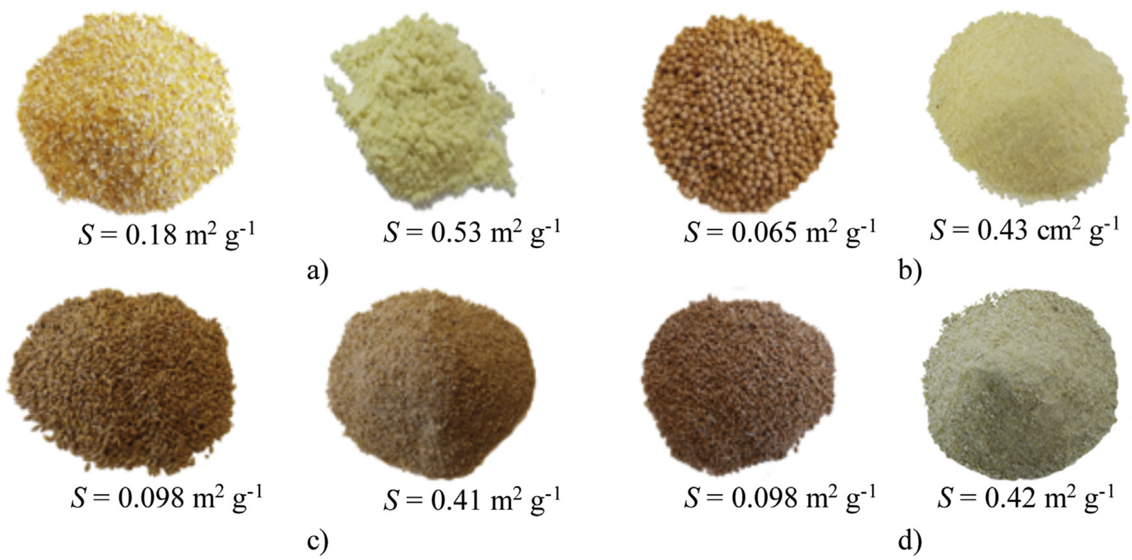

Fig. 12 – The obtained samples of the raw material before and after treatment: a) maize; b) peas; c) rye; d) wheat.

process of grinding of the loose raw material was carried out on the basis of the results of the analysis of the selected samples (Fig. 12).

After processing the results of the obtained data in the statistical environment Statistic 10.0 (Statsoft, Tulsa, OK, USA), dependencies of the evaluation criteria upon the investigated factors were obtained in the form of the second-order multiple regression equations, considering the most significant factors that determine the kinetics of the grinding process, the vibration acceleration factor a, m s^{-2}, the diameter of the balls d, mm; and the degree of loading of the grinding chambers E % with the technical filler. For the vibratory mill with angular oscillations, the regression models will have then the following forms, respectively for the specific surface area of the initial raw material S, m^2 g^{-1} (Eq. (13)) and the energy consumption of the vibrator N, kW h (Eq. (14)), and for the concerning indicators the following values: the multiple determination coefficient –$D = 0.82$; the coefficient of multiple correlation – $R = 0.90$; the criterion Fisher's – $F = 5.7$. Analysis of the above indicators confirms the high level of adequacy of the regression models.

$$S = -204 - 38.5\cdot a + 584.5\cdot d + 55\cdot E + 1\cdot a^2 - 80\cdot d^2 - 0.6\cdot E^2 + 4.1\cdot E\cdot d, \tag{13}$$

$$N = 987.3 - 19\cdot a + 122\cdot d - 6.6\cdot E - 0.3\cdot a^2 - 8.5\cdot d^2 + 0.7\cdot a\cdot d - 0.7\cdot d\cdot E. \tag{14}$$

By the results of experimental studies and on the basis of the constructed response surfaces of the obtained regression equations efficient parameters of their operation were determined. At the same time, a compromise value was found by the Cramer method in the Mathcad 15 (PTC Office, Moscow, Russia) mathematical environment. Thus, at the angular velocity of the drive shaft 110 rad s^{-1} and the moisture content of the material 8–11%, the rational values of the vibration acceleration of the grinding chamber are 45–50 m s^{-2}, the diameter of the holes in the separation surface is 0.5 mm; the specific area of the material is 0.5 m^2 g^{-1}; the productivity – 220 kg h^{-1}; energy consumption – 0.75 kW h, with the specific energy consumption being equal to 0.003 kW h kg^{-1}.

4. Conclusions

A mathematical model of a vibratory mill has been developed from the foundation of the theory of angular oscillations, which made it possible to obtain a functional dependence of the movement of its grinding chambers and to establish that the maximal dynamic state of the technological filler and the surface treatment is ensured at an angular velocity of the drive shaft of 120 rad s^{-1} and the angle of the vibration drive placement of 290°. With these parameters the ratio of the amplitude components of oscillations is 2.5 times.

The conclusions from these theoretical studies of the process of fine grinding bulk raw materials using a vibratory mill with angular vibrations was confirmed by experiment. It was established that at 120 rad s^{-1} and the angle of placement of the vibration drive equal to 290°, the limits of the rational operating parameters of the developed grinding machine will have the following values: vibro-acceleration 45–50 m s^{-2} the vibration intensity 16 m^2 s^{-3} the grinding chambers 50–60% with a diameter of the crushing balls 3–5 mm. It has also been established that these technical solutions will make it possible to produce a material with a specific surface of 5000 cm^2 g^{-1} at a rate of 220 kg h^{-1} and specific energy consumption of 0.003 kW h kg^{-1}.

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