VINNITSA NATIONAL AGRARIAN UNIVERSITY
Department of General Engineering Sciences and Labour Safety

# TRANSIENTS IN THE SIMPLEST ELECTRICAL CIRCUITS

by Associate Professor V. Hraniak

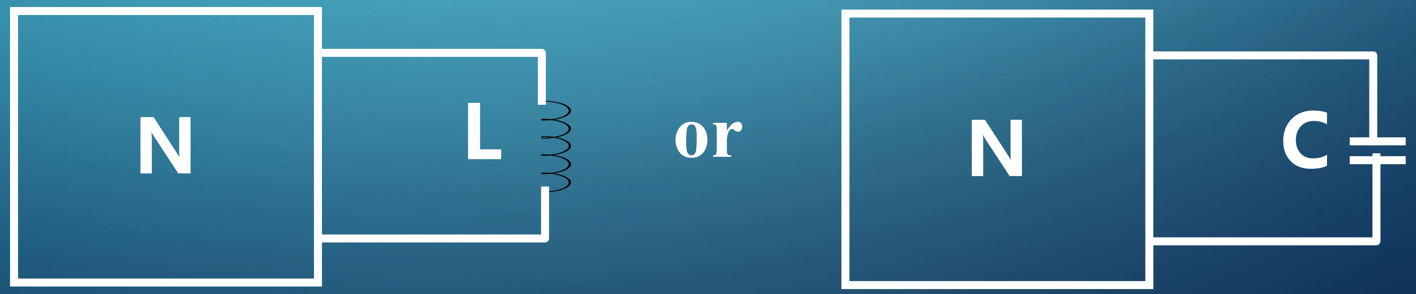
# FIRST-ORDER RC CIRCUITS

- **First-order circuit**

  **Only one (equivalent) capacitor or inductor is included in a linear circuit.**

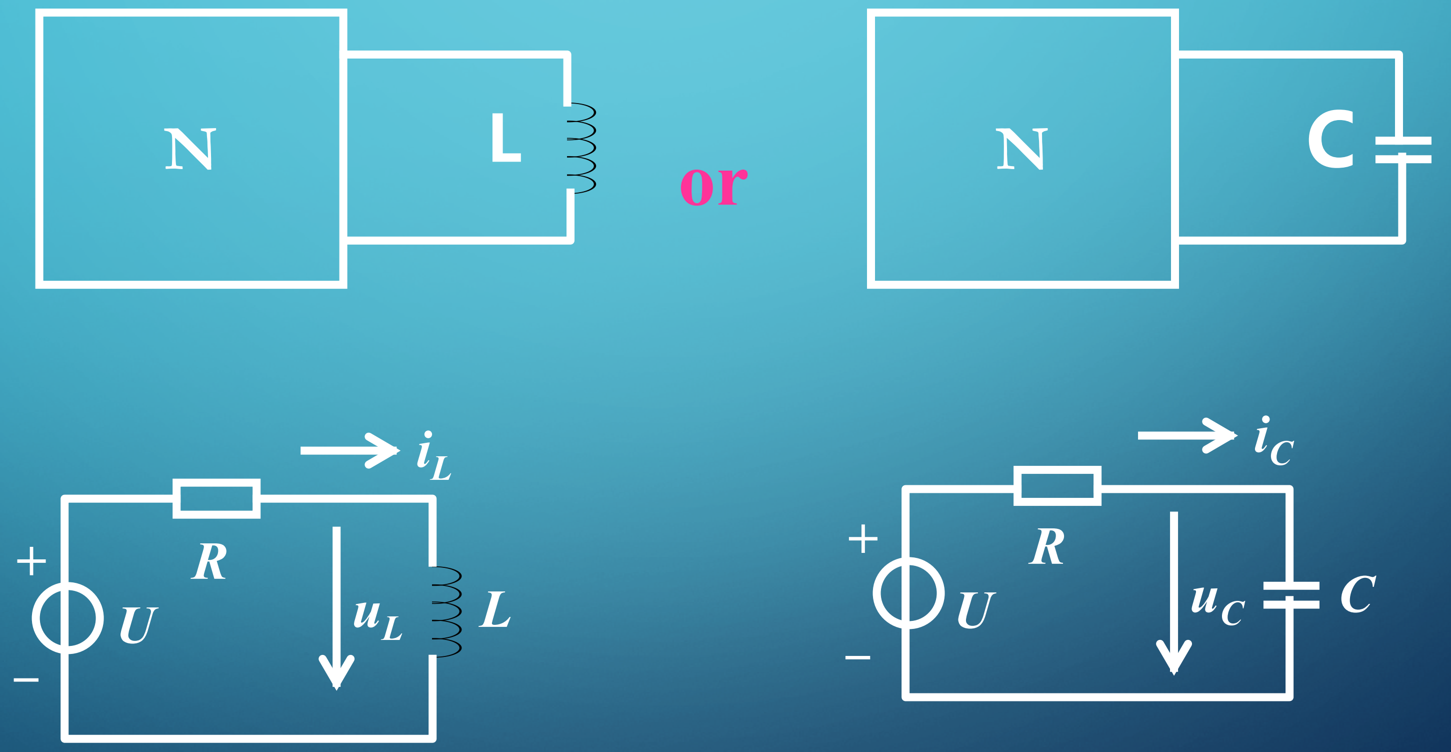
- **Equivalent circuit of First-order circuit**

  **Two parts: one (equivalent) capacitor or inductor; a two terminal network with resistance and sources.**

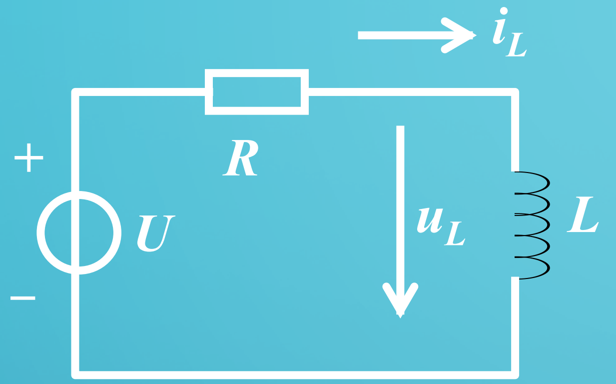


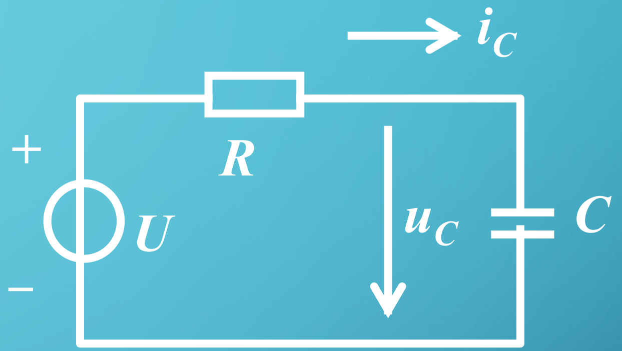
# First-order RC Circuits

- According to Thevenin Law



# Differential equation of first-order RC circuit




$$u_R + u_L = U$$

$$Ri_L(t) + L\frac{di_L(t)}{dt} = U$$

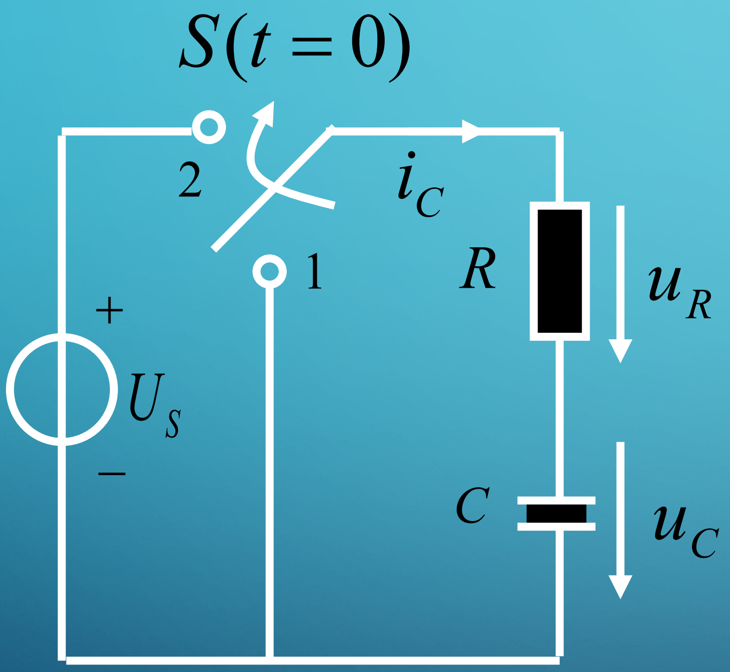
$$\frac{L}{R}\frac{di_L(t)}{dt} + i_L(t) = \frac{U}{R}$$

$$u_R + u_C = U$$

$$RC\frac{du_C}{dt} + u_C = U$$

# FIRST-ORDER RC CIRCUITS

- **Example: to find the transient response after changing circuit when** $t=0$.

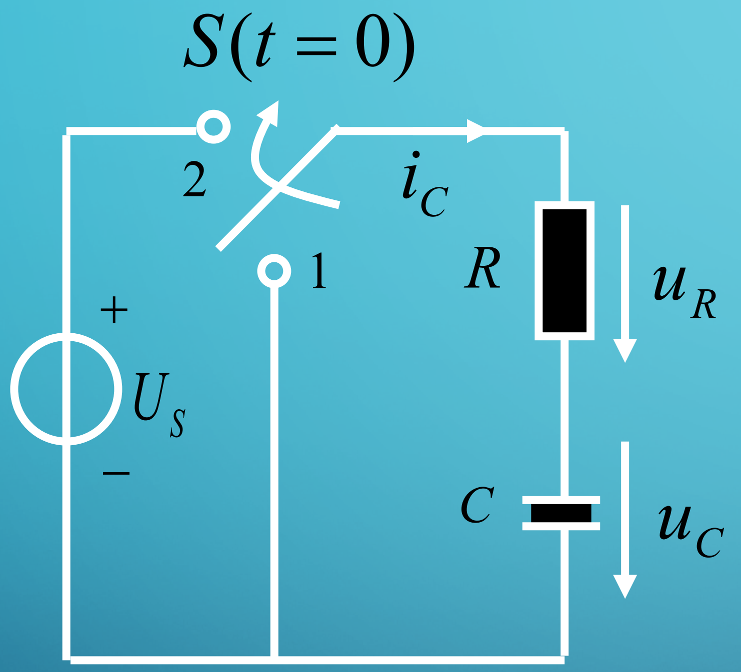



$$u_C(0-) = 0$$

**Solution:**

| $t$ \ $f(t)$ | $u_C$ | $u_R$ | $i$ |
|---|---|---|---|
| 0− | 0 | 0 | 0 |
| 0+ | 0 | $U_S$ | $\frac{U_S}{R}$ |
| ∞ | $U_S$ | 0 | 0 |

# First-order RC Circuits




$$u_R + u_C = U_S$$

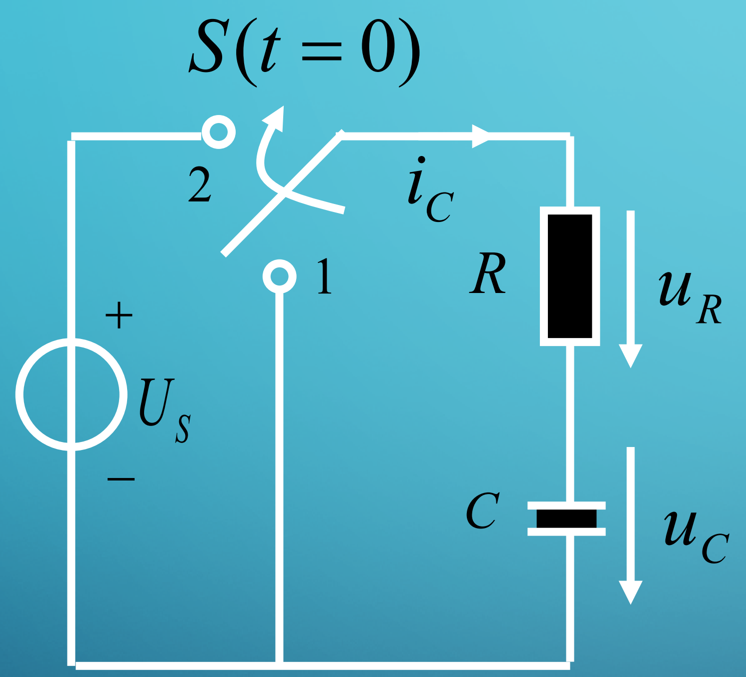
$$u_R = Ri \qquad i = C\frac{du_C}{dt}$$

$$RC\frac{du_C}{dt} + u_C = U_S$$

$$u_C(0-) = 0$$

$$u_C(0+) = u_C(0-) = 0$$

# First-order RC Circuits




$$RC\frac{du_C}{dt}+u_C=U_S$$

$$u_C=u_C^{'}+u_C^{''}$$

$$u_C^{'}=Ae^{st}$$

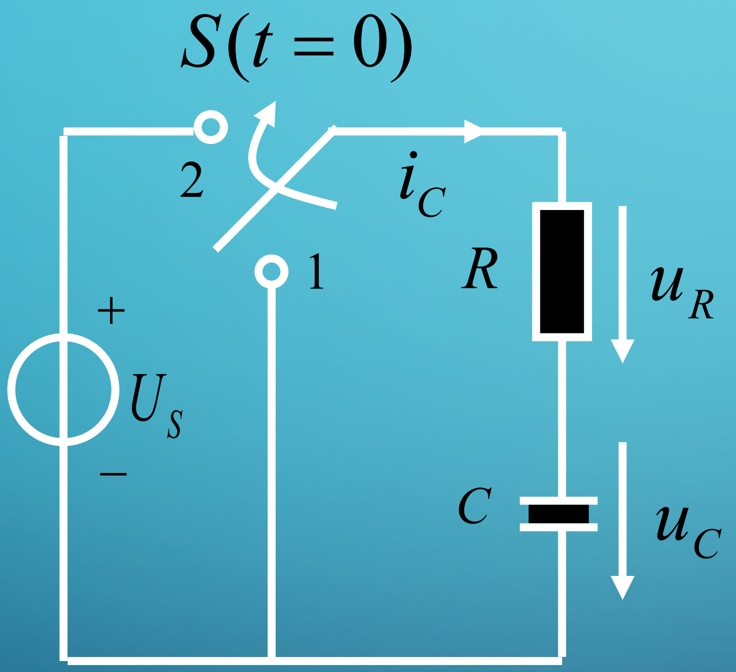
——homogeneous solution

$$u_C^{''}$$

——particular solution

# First-order RC Circuits

- **homogeneous solution**




$$RC\frac{du_C}{dt}+u_C=U_S$$

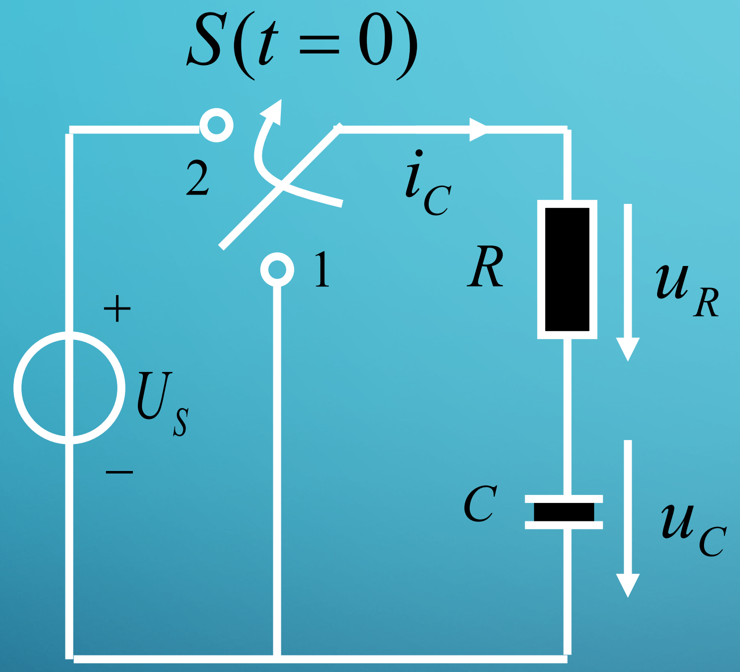
$$RCs+1=0$$

$$s=-\frac{1}{RC}$$

$$u_C'=Ae^{-\frac{1}{RC}t}$$

# First-order RC Circuits

- **Particular solution**




$$RC\frac{du_C}{dt}+u_C=U_S$$

**Therefore**

$$u_C''=u_C(\infty)=U_S$$

**Then, the final solution is**

$$u_C=u_C'+u_C''=Ae^{st}+U_S$$

# First-order RC Circuits

- **The solution of differential equation**

$$u_C = u_C^{'} + u_C^{''} = Ae^{st} + U_S$$

**Substituting the initial condition：**

$$u_C(0+) = u_C^{'} + u_C^{''} = Ae^{s0} + U_S = 0$$

$$A = u_C(0+) - u_C(\infty) = -U_S$$

$$u_C(t) = u_C(\infty) + [u_C(0+) - u_C(\infty)]e^{-\frac{1}{RC}t}$$

$$= U_S - U_S e^{-\frac{1}{RC}t}$$

# First-order RC Circuits

- **The solution of differential equation**

$$\tau = RC$$ ——Time constant

$$u_C(t) = u_C(\infty) + [u_C(0+) - u_C(\infty)]e^{-\frac{t}{\tau}}$$

$u_C(\infty)$ ——Steady state value

$u_C(0+)$ ——Initial value

# First-order RL Circuits

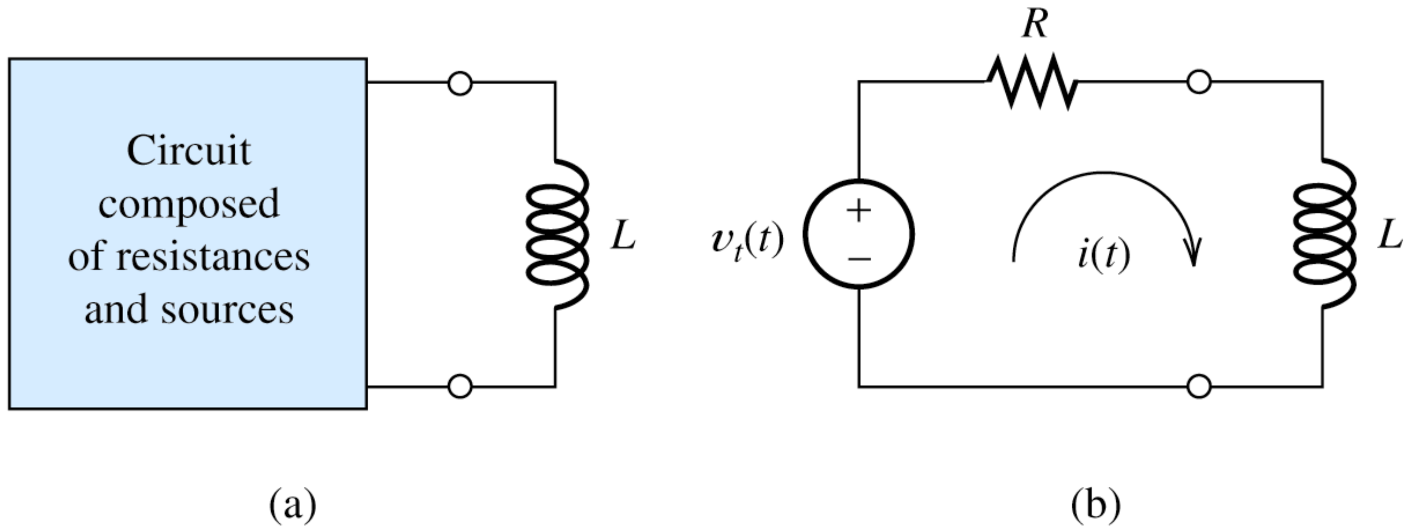
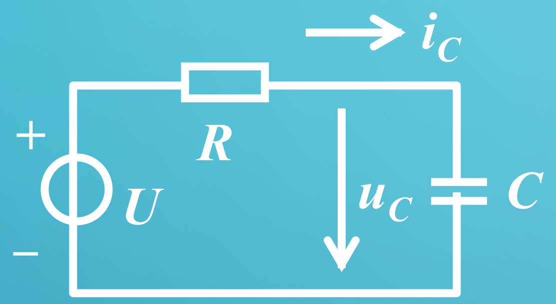



**Figure 4.13** A circuit consisting of sources, resistances, and one inductance has an equivalent circuit consisting of a voltage source and a resistance in series with the inductance.

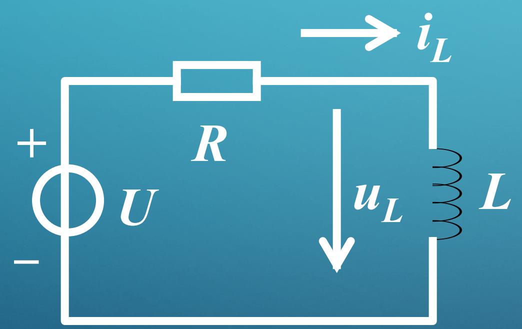
# First-order RL Circuits

- **Time constant**




$\tau = RC$

$$RC\frac{du_C}{dt} + u_C = U$$




$$\frac{L}{R}\frac{di_L(t)}{dt} + i_L(t) = \frac{U}{R}$$

$\tau = L/R$

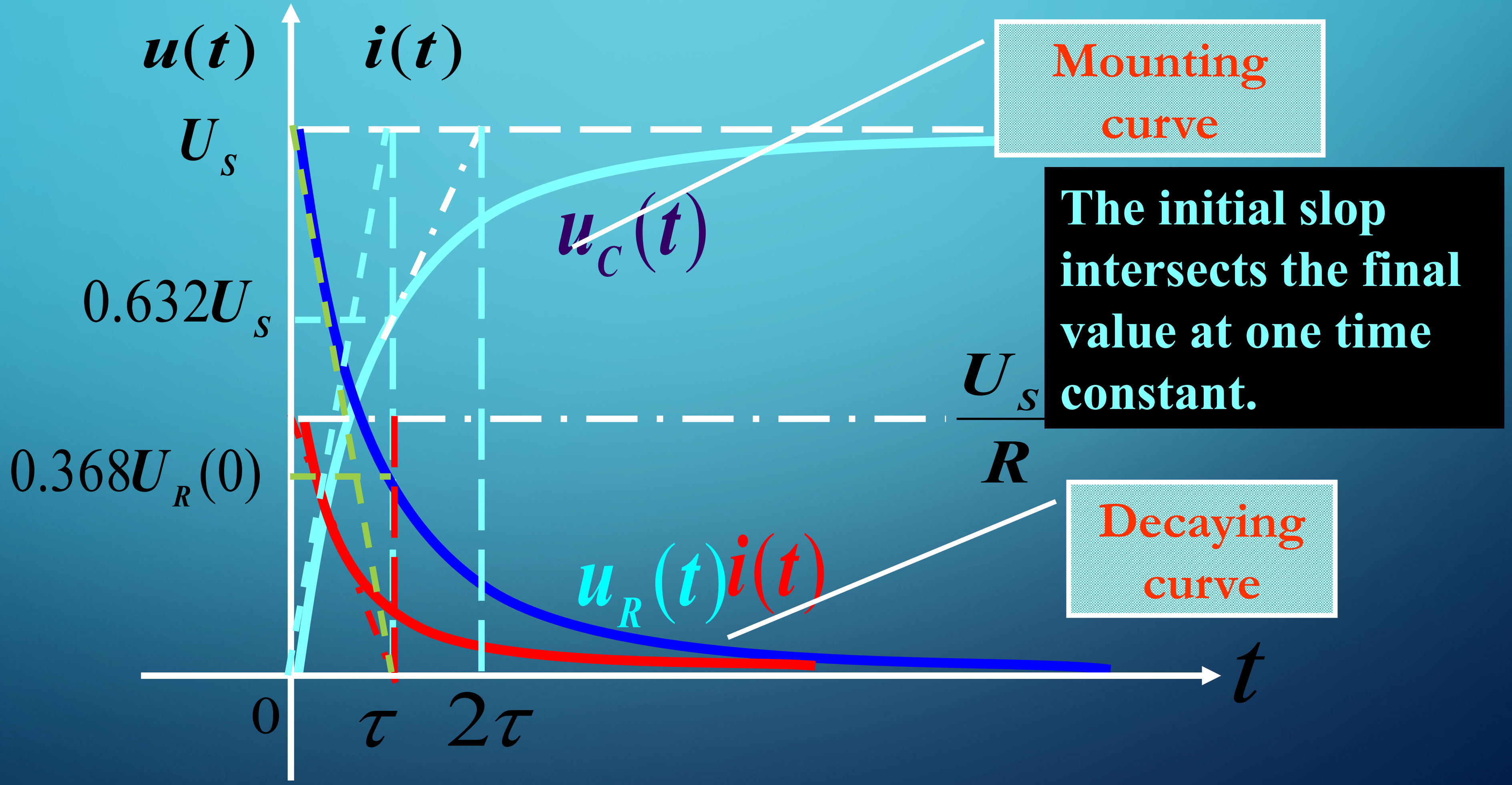
- **Time constant reflects the length of transient period.**

| $t$ | $\tau$ | $2\tau$ | $3\tau$ | $4\tau$ | $5\tau$ | $6\tau$ | $7\tau$ |
|---|---|---|---|---|---|---|---|
| $e^{-t/\tau}$ | 36.8% | 13.5% | 5% | 1.8% | 0.3% | 0.25% | 0.09% |

- **After one time constants, the transient response is equal to 36.8 percent of its initial value.**

- **After about five time constants, the transient response is over.**

- **Time constant reflects the length of transient period.**
  - The curves versus time



•**Example 4.2 Find voltage of *v*(*t*) and current *i*(*t*) in this circuit for *t*>0.**

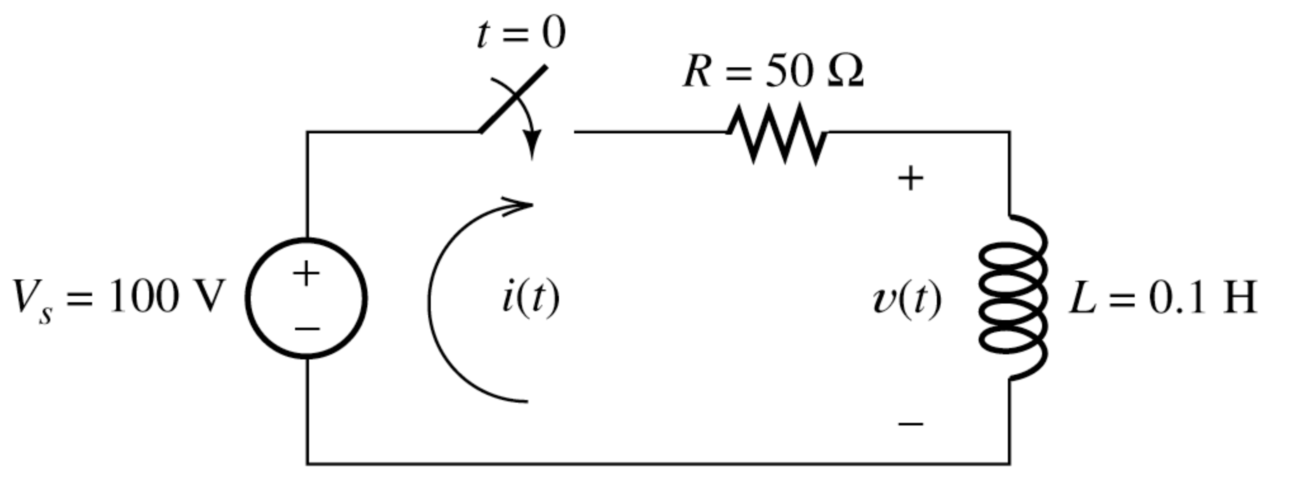



Figure 4.7 The circuit analyzed in Example 4.2.

**Answer:**

$$i(t) = 2 - 2e^{-\frac{t}{\tau}}(\mathrm{A}), v(t) = 100e^{-\frac{t}{\tau}}(\mathrm{V})$$

$$\tau = \frac{L}{R} = \frac{0.1}{50} = 2(\mathrm{ms})$$

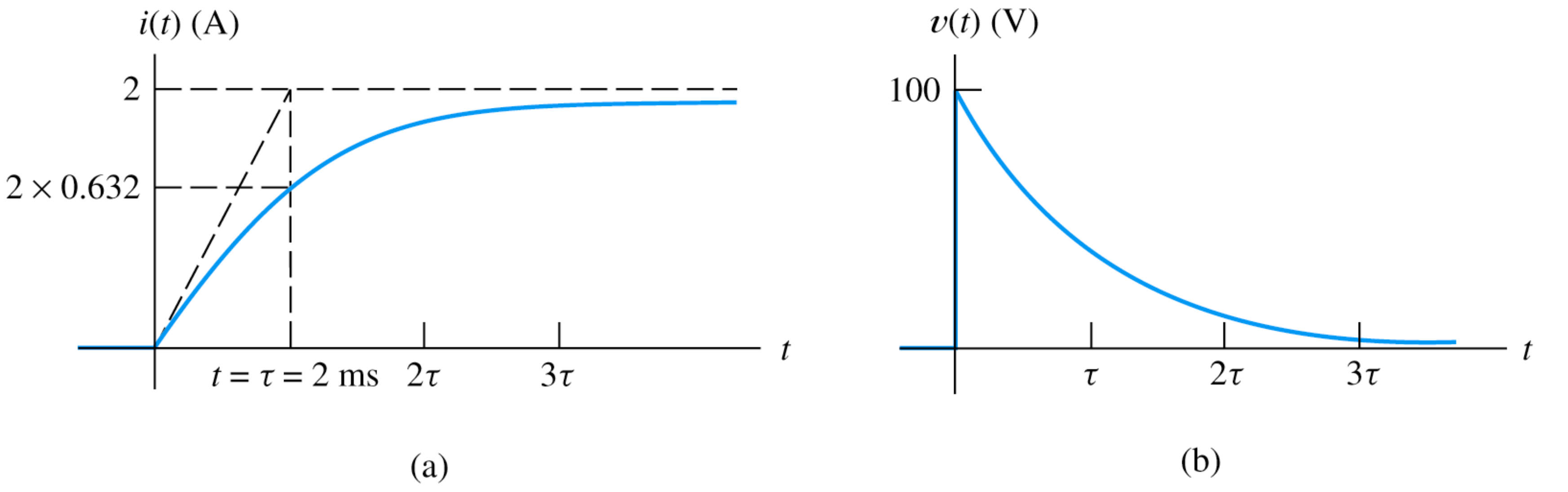



**Figure 4.8** Current and voltage versus time for the circuit of Figure 4.7.

$$i(t) = 2 - 2e^{-\frac{t}{\tau}}(\text{A}), v(t) = 100e^{-\frac{t}{\tau}}(\text{V})$$

$$\tau = \frac{L}{R} = \frac{0.1}{50} = 2(\text{ms})$$