VINNITSA NATIONAL AGRARIAN UNIVERSITY

Department of General Engineering Sciences and Labour Safety

OPERATOR METHOD OF CALCULATION OF TRANSIENTS

by Associate Professor V. Hraniak

OHM'S RULE IN OPERATOR FORM

For a circle R, L, C $$I(s)\cdot Z(s)=E(s)+Li(s)-\frac{u_c(0)}{s}$$

For zero initial conditions $$I(s)\cdot Z(s)=E(s)$$

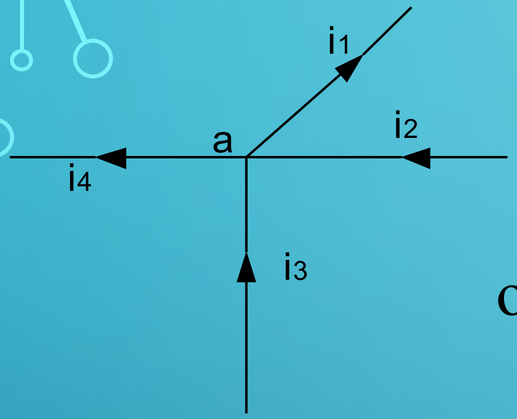
Thus, in a circle with zero initial conditions, Ohm's rule is also valid for images. If the initial conditions are not zero, then two more EMFs are added to the external EMF, which characterize the initial energy reserves in the electric field of the capacitor and the magnetic field of the inductor

$$\left(-\frac{u_{c(0)}}{s}\right) \quad \text{and} \quad Li_L(0)$$

So, if we take into account additional EMF in a circle, then also in this case Ohm's rule is valid for images.

KIRCHHOFF'S FIRST RULE IN OPERATOR FORM

For instantaneous values according to Kirchhoff's first rule



$$i_1 - i_2 - i_3 + i_4 = 0$$

In the operator form, taking into account the linearity property of the Laplace transform, we have:

$$I_1(s) - I_2(s) - I_3(s) + I_4(s) = 0$$

In general view, Kirchhoff's first rule

$$\sum_{k=1}^{n} i_k = 0$$

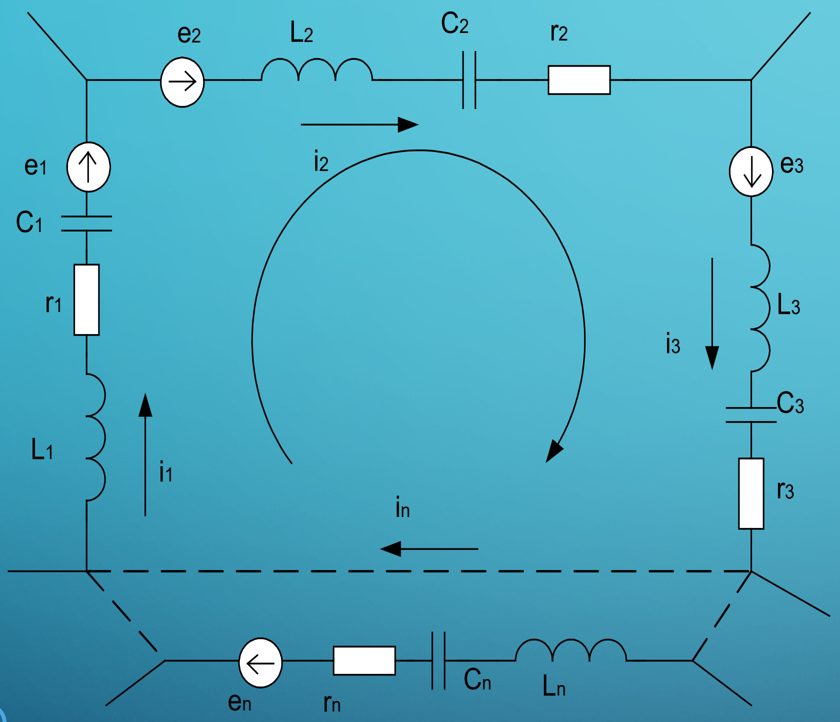
For originals

$$\sum_{k=1}^{n} I_k(s) = 0$$

For images

KIRCHHOFF'S SECOND RULE IN OPERATOR FORM

For a loop that contains n circuits, in each of which there is a source of EMF e_k, inductance L_k, and capacitance C_k and resistance R_k



For originals

$$\sum_{k=1}^{n}\left(L\frac{di_k}{dt}+r_k i_k+u_{ck}\right)=\sum_{k=1}^{n}e_k$$

For images

$$\sum_{k=1}^{n}I_k(s)Z_k(s)=\sum_{k=1}^{n}\left[E_k(s)+L_k i_k(s)-\frac{u_{ck}(0)}{s}\right]$$

$$\sum_{k=1}^{n}I_k(s)Z_k(s)=\sum_{k=1}^{n}E_k(s)$$

TRANSITION FROM THE IMAGE TO THE ORIGINAL

Inverse Laplace transform (general equation)

$$f(t) = \frac{1}{2\pi j} \int_{\delta - j\infty}^{\delta + j\infty} F(s) e^{st} ds$$

For the standard function we have:

$$\frac{A_k}{s - s_k} \xrightarrow{\cdot} A_k e^{s_k t}$$

TABLE 6.2.1 Elementary Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}, \quad s > 0$
2. e^{at}	$\frac{1}{s-a}, \quad s > a$
3. $t^n, \quad n =$ positive integer	$\frac{n!}{s^{n+1}}, \quad s > 0$
4. $t^p, \quad p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5. $\sin at$	$\frac{a}{s^2 + a^2}, \quad s > 0$
6. $\cos at$	$\frac{s}{s^2 + a^2}, \quad s > 0$
7. $\sinh at$	$\frac{a}{s^2 - a^2}, \quad s > \lvert a \rvert$
8. $\cosh at$	$\frac{s}{s^2 - a^2}, \quad s > \lvert a \rvert$
9. $e^{at} \sin bt$	$\frac{b}{(s-a)^2 + b^2}, \quad s > a$
10. $e^{at} \cos bt$	$\frac{s-a}{(s-a)^2 + b^2}, \quad s > a$
11. $t^n e^{at}, \quad n =$ positive integer	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
12. $u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$
13. $u_c(t) f(t-c)$	$e^{-cs} F(s)$
14. $e^{ct} f(t)$	$F(s-c)$
15. $f(ct)$	$\frac{1}{c} F\left(\frac{s}{c}\right), \quad c > 0$
16. $\int_0^t f(t-\tau) g(\tau)\, d\tau$	$F(s) G(s)$
17. $\delta(t-c)$	e^{-cs}
18. $f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - \cdots - f^{(n-1)}(0)$
19. $(-t)^n f(t)$	$F^{(n)}(s)$

DECOMPOSITION THEOREM

A proper rational fraction can be represented as a finite sum of simple fractions

Let's present the operator images of the sought functions in the form of the ratio of two polynomials

$$F(s)=\frac{M(s)}{N(s)}=\frac{a_m s^m + a_{m-1}s^{m-1}+\ldots+a_1 s^1 + a_0}{b_n s^n + b_{n-1}s^{n-1}+\ldots+b_1 s^1 + s_0}$$

In this case, this expression can be written as a sum of simple fractions

$$F(s)=\frac{M(s)}{N(s)}=\frac{A_1}{s-s_1}+\frac{A_2}{s-s_2}+\ldots+\frac{A_k}{s-s_k}+\ldots+\frac{A_n}{s-s_n}=\sum_{k=1}^{n}\frac{A_k}{s-s_k}$$

where s_k - the roots of the denominator

A_k - unknown constants.

Therefore, the main task of finding the original f(t) is to determine the coefficients A_k.

To find the values of A_k, multiply the right and left parts by $(s - s_k)$.

$$\frac{M(s)}{N(s)}(s-s_k)=A_1\frac{s-s_k}{s-s_1}+A_2\frac{s-s_k}{s-s_2}+\ldots+A_k+\ldots+A_n\frac{s-s_k}{s-s_n}$$

and direct s to s_k

$$A_k=\lim_{s\to s_K}\frac{M(s-s_k)}{N(s)}$$

We will reveal the uncertainty according to Lopital's rule, that is, we will differentiate separately the numerator and denominator of the expression, which is under the sign of the limit (lim), and substitute $s=s_k$

$$A_k=\frac{P(s_k)}{Q'(s_k)}$$

After performing the substitution, we get

$$F(s)=\sum_{k=1}^{n}\frac{P(s_k)}{Q'(s_k)}\frac{1}{s-s_k}$$

Then the original can be identified

$$f(t)=\sum_{k=1}^{m}\frac{P(s_k)}{Q'(s_k)}e^{s_k t}.$$

where

s_k – the root of the denominator;

n – number of roots of the denominator.

EXAMPLE

Find the original by a known image

$$F(s)=\frac{s^2+4s+8}{s\left(s^2+6s+8\right)}$$

Let's find the roots of the denominator, equating it to zero

$$s\left(s^2+6s+8\right)=0$$

$$s_1=0$$

$$s_{2,3}=\frac{-6\pm\sqrt{36-32}}{2}=\frac{-6\pm2}{2} \qquad s_2=-2 \qquad s_3=-4$$

Let’s determine the value of the numerator at the calculated values of s

$$M(s_1)=0^2+4\cdot0+8=8$$

$$M(s_2)=\left(-2\right)^2+4\cdot\left(-2\right)+8=4$$

$$M(s_3)=\left(-4\right)^2+4\cdot\left(-4\right)+8=8$$

Let’s find the derivative of the denominator

$$N'(p)=\left(p^2+6p+8\right)+p\cdot(2p+6)=3p^2+12p+8$$

The value of the derivative at the calculated values of the roots of the denominator

$$N'(p_1)=0^2+0\cdot 6+8+0\cdot(2\cdot 0+6)=8$$

$$N'(p_2)=(-2)^2+(-2)\cdot 6+8+(-2)\cdot[2\cdot(-2)+6]=-4$$

$$N'(p_3)=(-4)^2+(-4)\cdot 6+8+(-4)\cdot[2\cdot(-4)+6]=8$$

Then by the decomposition theorem

$$f(t)=\frac{8}{8}\cdot e^{0\cdot t}+\frac{4}{-4}\cdot e^{-2\cdot t}+\frac{8}{8}\cdot e^{-4\cdot t}=1-e^{-2\cdot t}+e^{-4\cdot t}$$

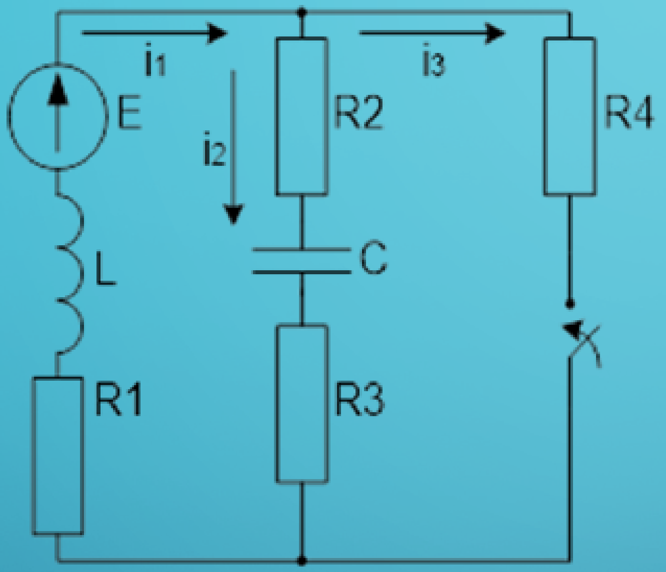
INDEPENDENT WORK

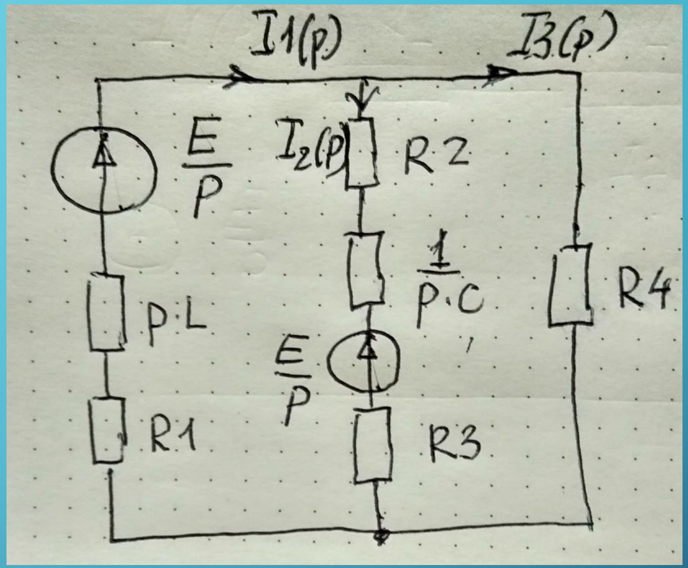
Find the original by a known image

$$F(s) = \frac{s^2 + 4s + 8}{s(s^2 + 6s + 8)}$$

EXAMPLE

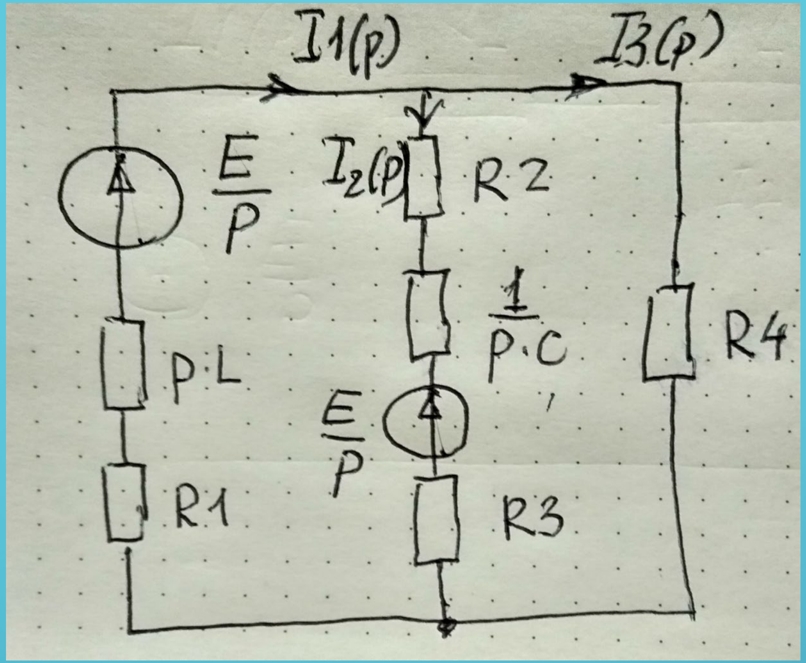
Calculate the transient in an electric circuit by operator method





Incoming data

$L := 1 \cdot 10^{-3}$	$C := 10^{-5}$	$R1 := 12$	$R2 := 12$
$E := 200$		$R4 := 10$	$R3 := 10$



System of equations in operator form

$$I11 - I2 - I3 = 0$$

$$I11 \cdot R1 + I11 \cdot p \cdot L + I2 \cdot R2 + I2 \cdot R3 + I2 \cdot \frac{1}{p \cdot C} = 0$$

$$I3 \cdot R4 - I2 \cdot R2 - I2 \cdot R3 - I2 \cdot \frac{1}{p \cdot C} = \frac{E}{p}$$

Solution of the system of equations

$$\mathrm{Find}(I11, I2, I3) \rightarrow \begin{bmatrix} 12500 \cdot \dfrac{(11 \cdot p + 50000)}{\left(22000 \cdot p + 68750000 + p^2\right) \cdot p} \\ \dfrac{-25}{4} \cdot \dfrac{(12000 + p)}{\left(22000 \cdot p + 68750000 + p^2\right)} \\ \dfrac{25}{4} \cdot \dfrac{\left(34000 \cdot p + 100000000 + p^2\right)}{\left(22000 \cdot p + 68750000 + p^2\right) \cdot p} \end{bmatrix}$$