

VINNITSA NATIONAL AGRARIAN UNIVERSITY

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COMPLEX SYMBOLIC METHOD OF CALCULATING ALTERNATING CURRENT ELECTRIC CIRCUITS

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THE ESSENCE OF THE COMPLEX SYMBOLIC CALCULATION METHOD

The complex symbolic method of analyzing sinusoidal current circuits is based on the existence of a mutually unambiguous correspondence between trigonometric functions and complex numbers, which makes it possible to conduct calculations not with sinusoidal functions of currents and voltages, but with their complex images.

COMPLEX IMAGES OF ELECTRICAL PARAMETERS

In the future, we will conduct the calculation of sinusoidal current circles in effective (RMS) values, therefore we will show the correspondence of complex effective values of current and voltage to their sinusoidal functions.

$$u = U_m \sin(\omega t + \beta) \rightarrow \underline{U} = \frac{U_m}{\sqrt{2}} e^{j\beta},$$

$$i = I_m \sin(\omega t + \alpha) \rightarrow \underline{I} = \frac{I_m}{\sqrt{2}} e^{j\alpha}.$$

INDIVIDUAL WORK

Represent instantaneous currents in effective complex form:

$$i(t) = 10 \cdot \sin(\omega t - 25^\circ) (A)$$

$$i(t) = 8 \cdot \sin(\omega t + 360^\circ) (A)$$

$$i(t) = 15 \cdot \cos(\omega t) (A)$$

$$i(t) = -10 \cdot \sin(\omega t + 45^\circ) (A)$$

INDIVIDUAL WORK

According to the given complex effective values of the currents, write their instantaneous values:

$$\underline{I} = 6e^{-j45^\circ}$$

$$\underline{I} = 4e^{j120^\circ}$$

$$\underline{I} = 12e^{j21^\circ}$$

Complex images of active, reactive and total circuit resistances are defined as:

- complex resistance of the resistor r is a real number and is equal to r ;
- complex resistance of the inductance L is an imaginary number and is equal to

$$j\omega L = jx_L$$

- complex resistance of the capacitor C is an imaginary number and is equal to

$$\frac{1}{j\omega C} = -\frac{j}{\omega C} = -jx_C$$

- full complex resistance of the circuit section rLC is a complex number and is written in algebraic form as

$$\underline{Z} = r + j\omega L + \frac{1}{j\omega C} = r + j\left(\omega L - \frac{1}{\omega C}\right) = r + j(x_L - x_C) = r + jx$$

The concept of complete complex conductance is introduced as

$$\underline{Y} = \frac{1}{\underline{Z}} = \frac{1}{r + jx} = \frac{r - jx}{r^2 + x^2} = \frac{r}{r^2 + x^2} - j \frac{x}{r^2 + x^2}$$

The values

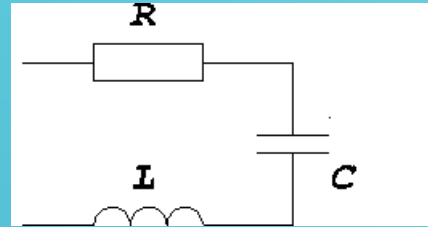
$$g = \frac{r}{r^2 + x^2}$$

$$b = \frac{x}{r^2 + x^2}$$

are called active and reactive conductivity, respectively.

EXAMPLE

Calculate the complex equivalent resistance and complex equivalent conductance for the given circuit



$$R = 20 \text{ Ohm}$$

$$L = 100 \text{ mH}$$

$$C = 190 \text{ } \mu\text{F}$$

$$f = 50 \text{ Hz}$$

CALCULATION

$$\omega = 2 \cdot \pi \cdot f = 2 \cdot 3,14 \cdot 50 = 314 \text{ (rad / s)}$$

$$X_L = \omega \cdot L = 314 \cdot 100 \cdot 10^{-3} = 31,4 \text{ (Ohm)} \quad \underline{X_L} = 31,4 \cdot j \text{ (Ohm)}$$

$$X_C = \frac{1}{\omega \cdot C} = \frac{1}{314 \cdot 190 \cdot 10^{-6}} = 16,76 \text{ (Ohm)} \quad \underline{X_C} = -16,76 \cdot j \text{ (Ohm)}$$

$$\underline{Z} = R + \underline{X_C} + \underline{X_L} = 20 - 16,76 \cdot j + 31,4 \cdot j = 20 + 14,64 \cdot j \text{ (Ohm)}$$

$$\underline{Y} = \frac{1}{\underline{Z}} = \frac{1}{20 + 14,64 \cdot j} = \frac{20 - 14,64 \cdot j}{400 + 214,33} = 0,033 - 0,024 \cdot j \text{ (S)}$$

RULES OF ELECTRICAL ENGINEERING IN THE COMPLEX FORM

1. Ohm's rule

$$\underline{U} = \underline{I} \underline{Z}$$

2. First Kirchhoff's rule

$$\sum_{k=1}^n \underline{I}_k = 0$$

3. Second Kirchhoff's rule

$$\sum_{k=1}^n \underline{E}_k = \sum_{k=1}^n \underline{I}_k \underline{Z}_k$$

POWER IN SINUSOIDAL CURRENT CIRCUITS

Let a current flow and an applied voltage in some linear electric circuit

$$i(t) = I_m \sin(\omega t + \alpha)$$

$$u(t) = U_m \sin(\omega t + \beta)$$

The active power of the circuit is found as:

$$P = UI \cos \varphi$$

or

$$P = I^2 r$$

To characterize energy sources, the concept of full power is introduced, which is measured in volt-amperes [VA]. This is the maximum power that the source can give:

$$S = UI$$

or

$$S = I^2 Z$$

Another concept is introduced - reactive power, which characterizes the exchange of energy between the consumer and the source.

$$Q = UI \sin \varphi$$

or

$$Q = I^2 X$$

The unit of measurement of reactive power is volt-ampere reactive [var].

POWER BALANCE IN ALTERNATING CURRENT CIRCUITS

It follows from the law of conservation of energy that in an electric circuit all the power enters the circuit and at any moment of time is equal to all the power absorbed by the circuit. This is true for both active and reactive power.

Thus, the total active power of energy sources in any circuit is equal to the consumed active power of this circuit. The sum of the reactive powers given by the sources is equal to the sum of the reactive powers of the receivers. This condition is called the balance of active and reactive power, respectively. Mathematically, this is written as:

$$\sum_k U_k I_k \cos \phi_k = \sum_k I_k^2 R_k,$$

$$\sum_k U_k I_k \sin \phi_k = \sum_k I_k^2 X_k.$$

The background is a blue gradient with white circuit-like lines in the corners. The lines consist of straight segments and small circles, resembling a printed circuit board or a network diagram. They are located in the top-left, top-right, bottom-left, and bottom-right corners.

THANK FOR YOUR ATTENTION!